Math 322 Assignment 1, Spring 2013 **Due at 3 PM Wednesday, April 3rd**

1. Solve the following two initial value problems (IVPs) involving second order, linear, constant coefficient ordinary differential equations (ODEs). Assume that the independent variable is t.

   (a) $y'' + 5y' + 6y = 0, \quad y(0) = 4,\ y'(0) = -1$ (b) $y'' + 4y = 0, \quad y(0) = -2,\ y'(0) = 1$

   There are some resources listed under *Read* and *Watch* in the schedule, under today's date, that can help you with this. Be prepared to offer your solution to either of these at the next class meeting.

2. (a) Using the power series for e^x, use substitution to obtain the power series of e^{-2x}. Give your answer as the first five terms expanded out (followed by $+\cdots$) *AND* in summation notation as a series in powers of x.

   (b) Repeat part (a) with the series for the sine function to obtain a series for $\sin 4x$.

3. In this exercise we'll obtain the solution to the IVP from Exercise 1(b) using power series. Let $y = a_0 + a_1x + a_2x^2 + a_3x + ...$

   (a) Give the power series for y' in expanded form, including the first five terms.

   (b) Give the power series for y'' in expanded form, including the first five terms.

   (c) Give the power series for $y'' + 4y$ by
   - distributing 4 into the first seven terms of the series for y and adding the series for y'' to that, and
   - combining terms with like powers of x.

   (d) In order for the series you obtained in (c) to be zero (meaning for all values of x), each coefficient must be zero. Set the constant (first) term equal to zero and solve for a_2 in terms of a_0. Then set the coefficient of x equal to zero and solve for a_3 in terms of a_1.

   (e) When you set the coefficient x^2 equal to zero you'll solve for a_4 in terms of a_2. But you know a_2 in terms of a_0, so you can can substitute that in to get a_4 in terms of a_0. In the same way you can find a_5 in terms of a_1. Continue in this manner to get a_6 and a_7 in terms of a_0 and a_1.

   (f) Substitute the coefficients a_2, a_3,...,a_7 into the series $y = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + \cdots$. Then group all the terms with a_0 together, and group the ones with a_1 together. Factor out the a_0 and a_1.

   (g) You probably won't recognize the two series obtained when a_0 and a_1 are factored out. Multiply the numerator and denominator of each fraction so that the denominators are the factorial of the corresponding exponent of x.

   (h) **At this point you may stop if you wish**, and we'll finish it together in class. You may recognize the first series; try putting each numerator in the form $(bx)^n$ for some number b that is the same for each term. For the second series, you can multiply each term by two if at the same time you divide a_1 by two. Then try doing the same as was just suggested for the first series.