Math 322 ASSIGNMENT 15, SPRING 2013 **Due at 3 PM Thursday May 9th**

1. Solve the system $\mathbf{x}' = \begin{bmatrix} -2 & -2 \\ -\frac{1}{2} & -2 \end{bmatrix} \mathbf{x}$. Give your solution as two separate equations.

2. In this exercise you will plot what we will call **trajectories** or **solutions curves** for the solution to the system from Exercise 1. Note that your solution consists of two parts, $x_1 = x_1(t)$ and $x_2 = x_2(t)$. If we fix values for the two constants and plot pairs (x_1, x_2) at various increasing times we obtain a trajectory of the solution for those values of the constants. If we then change the values of the constants we will get a different trajectory. Each of these trajectories corresponds to different initial conditions.

   (a) Open the 3D Calc Plotter. I sent you a link, or you can just do a search for "3D calc plotter."

   (b) Click the smaller *Graph* and clear all graphs. Then click the smaller *Graph* again and select *Add a space curve.*

   (c) Enter your x_1 and x_2 **with the values of one for each of your constants** as x and y, and enter zero for z. Check the box at the bottom for *Restrict view to 2D.*

   (d) Set the time for $0 \le t \le 3$ and the number of steps for 300. Click the *View* tab at the top and make sure that only *Show point on curve* is checked.

   (e) Click *Animate* and you will see the trajectory, with a point traveling it as time goes on. Sketch the trajectory on a **large** graph, and label the starting point of the trajectory with $\mathbf{x}(0) =$ followed by the initial value vector. Put a couple of arrows on the trajectory to indicate the direction of travel as time increases.

   (f) Repeat the above for all of the following combinations: both constants negative one, one positive one and the other negative one and vice-versa, one constant one and the other zero and vice versa, one constant negative one and the other zero and vice versa. When you get done you should have eight trajectories

In Assignment 9 you solved an initial value problem involving a system $\mathbf{x}' = A\mathbf{x}$ where the eigenvalues and eigenvectors of A were complex. You found that in the end you were able to obtain solutions with only real numbers, but the process of getting there was a bit tedious. We'll present here a method for avoiding at least some of the computations that were involved.

Suppose that $\mathbf{x}' = A\mathbf{x}$, where A is a 2×2 matrix with complex eigenvalue $\alpha + \beta i$ having corresponding eigenvector $\mathbf{k}$. Let $\mathbf{b}_1$ be the real part of $\mathbf{k}$ and $\mathbf{b}_2$ be the imaginary part. Then the solution to the system is

$$\mathbf{x} = [c_1(\mathbf{b}_1 \cos \beta t - \mathbf{b}_2 \sin \beta t) + c_2(\mathbf{b}_1 \sin \beta t + \mathbf{b}_2 \cos \beta t)]e^{\alpha t} \tag{1}$$

3. Use the above method to solve the IVP $\begin{array}{rcl} x_1' &=& x_1 - x_2 \\ x_2' &=& 5x_1 - 3x_2 \end{array}$, $x_1(0) = -2,\ x_2(0) = 1$. Use the following steps:

   - Write the system in the form $\mathbf{x}' = A\mathbf{x}$ and give the initial values as a vector.
   - Find one eigenvalue and the corresponding eigenvector. Do this by hand or using some tool, but give the eigenvector without fractions. (Remember that any scalar multiple of an eigenvector is also an eigenvector, with the same eigenvalue.)
   - Use the method described above to obtain a solution of the form (1).
   - Apply the initial value to obtain c_1 and c_2.
   - Substitute c_1 and c_2 in, combine like terms and write the solution as two separate equations, one for x_1 and one for x_2.