Math 322 ASSIGNMENT 16, SPRING 2013 **Due at 3 PM Friday May 10th**

1. For each of the following systems, draw a phase portrait illustrating the behavior of the solutions, **using the phase plane plotter that can be found at the class web page**. Be sure to

   - include any trajectories along eigenvectors, which you may just have to put in "by hand"
   - include enough trajectories to give a good idea what is going on in all parts of the phase plane
   - put an arrowhead or two on each trajectory to indicate direction as tie increases

   In each case, try to predict what the phase portrait will look like before finding it.

   (a) $\mathbf{x}' = \begin{bmatrix} -2 & -2 \\ -\frac{1}{2} & -2 \end{bmatrix}\mathbf{x}$ $\quad \lambda = -3,\ -1,\ \mathbf{k} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\quad \mathbf{x} = c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix}e^{-3t} + c_2\begin{bmatrix} -2 \\ 1 \end{bmatrix}e^{-t}$

   (b) $\mathbf{x}' = \begin{bmatrix} 2 & 2 \\ \frac{1}{2} & 2 \end{bmatrix}\mathbf{x}$ $\quad \lambda = 3,\ 1,\ \mathbf{k} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\quad \mathbf{x} = c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix}e^{3t} + c_2\begin{bmatrix} -2 \\ 1 \end{bmatrix}e^{t}$

   (c) $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}\mathbf{x}$ $\quad \lambda = 4,\ -1,\ \mathbf{k} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad \mathbf{x} = c_1\begin{bmatrix} 2 \\ 3 \end{bmatrix}e^{4t} + c_2\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{-t}$

   (d) $\mathbf{x}' = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\mathbf{x}$ $\quad \lambda = 3,\ \mathbf{k} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\quad \mathbf{x} = c_1\begin{bmatrix} 2 \\ 1 \end{bmatrix}e^{3t} + c_2\begin{bmatrix} -2 \\ 1 \end{bmatrix}e^{3t}$

   (e) $\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix}\mathbf{x}$ $\quad \lambda = 3,\ \mathbf{k} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad \mathbf{x} = c_1\begin{bmatrix} -1 \\ 1 \end{bmatrix}e^{3t} + c_2\left\{\begin{bmatrix} -1 \\ 1 \end{bmatrix}te^{3t} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}e^{3t}\right\}$

   (f) $\mathbf{x}' = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}\mathbf{x}$ $\quad \lambda = 5 \pm 2i,\ \mathbf{k} = \begin{bmatrix} 1 \\ 1 \mp 2i \end{bmatrix}$ $\quad \mathbf{x} = [c_1(\mathbf{b}_1\cos 2t - \mathbf{b}_2\sin 2t) + c_2(\mathbf{b}_1\sin 2t + \mathbf{b}_2\cos 2t)]e^{5t}$

   (g) $\mathbf{x}' = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}\mathbf{x}$ $\quad \lambda = \pm 2i,\ \mathbf{k} = \begin{bmatrix} 2 \pm 2i \\ -1 \end{bmatrix}$ $\quad \mathbf{x} = [c_1(\mathbf{b}_1\cos 2t - \mathbf{b}_2\sin 2t) + c_2(\mathbf{b}_1\sin 2t + \mathbf{b}_2\cos 2t)]$

2. Consider the system $\mathbf{x}' = \begin{bmatrix} -3 & 1 \\ 2 & 1 \end{bmatrix}\mathbf{x}$.

   (a) Give $\left.\dfrac{dx_1}{dt}\right|_{(1,1)}$ and $\left.\dfrac{dx_2}{dt}\right|_{(1,1)}$. Then give $\left.\dfrac{dx_2}{dx_1}\right|_{(1,1)}$, using the fact that $\dfrac{dx_2}{dx_1} = \dfrac{dx_2/dt}{dx_1/dt}$.

   (b) Find $\dfrac{dx_2}{dx_1}$ at $(1,2)$, $(2,1)$ and $(0,1)$, labeling your answers clearly.

   (c) Plot a direction field at the points for which you have found slopes.