Math 322 Assignment 18, Spring 2013 **Due at 3 PM Thursday May 16th**

1. Use the Laplace transform to solve the initial value problem $y'' + 4y' + 8y = \sin 5t, \quad y(0) = 1, \; y'(0) = -2$. Many of you were a bit careless with grouping around the transform of the first derivative last time we did a Laplace problem - be careful about that! As we had been doing, convert the equation into the s variable using the tables, solve for $Y(s)$ and use Wolfram to do the inverse transform.

2. Write each of the following initial value problems for systems of equations in the form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}, \quad \mathbf{x}(0) = \mathbf{c}$.

(a)
$$\begin{aligned} x_1' - 2x_2 &= 3 \qquad & x_1(0) &= 0 \\ x_2' - x_1 + x_2 &= -t^2 \qquad & x_2(0) &= -1 \end{aligned}$$

(b)
$$\begin{aligned} x_1' + 3x_1 - 5x_2 &= e^{-t} \qquad & x_1(0) &= 2 \\ x_2' - 4x_1 + 3x_2 &= 3e^{-t} \qquad & x_2(0) &= 1 \end{aligned}$$

(c)
$$\begin{aligned} x_1' + x_1 - 3x_2 + 2x_3 &= 0 \qquad & x_1(0) &= 3 \\ x_2' + 2x_1 + x_2 - 4x_3 &= 3 \qquad & x_2(0) &= -1 \\ x_3' - x_1 + 5x_2 + 2x_3 &= \sin 3t \qquad & x_3(0) &= 2 \end{aligned}$$

(d)
$$\begin{aligned} x_1' + 5x_2 - 2x_3 &= \sin t \qquad & x_1(0) &= 0 \\ x_2' - x_1 + 2x_3 &= 0 \qquad & x_2(0) &= 1 \\ x_3' - 2x_1 - 3x_2 &= \cos t \qquad & x_3(0) &= 0 \end{aligned}$$

There is another exercise on the back.

3. Convert the ODE $x^{(4)} + 5x^{(3)} - 7x'' + 3x' - 4x = 0$ (here $x^{(4)}$ means the fourth derivative of x) into a system of first order equations as follows.

(a) Solve the ODE for $x^{(4)}$.

(b) Let $x_1 = x$, $x_2 = x'$, on up until you have renamed $x^{(3)}$. **Write these down, including the ones I've already given you**.

(c) Substitute these into your result for (a), then rearrange the right side so that the indices of x are increasing. **You are not replacing $x^{(4)}$ yet!**

(d) Now $x^{(4)}$ is the first derivative of one of your x_k. Figure out which one, take its derivative and substitute for $x^{(4)}$.

(e) You now have equations for each of x_1' through x_4', scattered about. Write them all as a list now.

(f) As done in Exercise 2, write your result from (e) in the form $\mathbf{x}' = A\mathbf{x}$. The matrix A will have lots of zeros and several ones in it.