Math 322 ASSIGNMENT 21, SPRING 2013 **Not to turn in - do for Wednesday, May 22nd**

The final answers to Exercises 2 and 3 are given below. I'll post the full solutions by the evening of 2/21.

1. Write the ODE $y^{(5)} - 2y^{(4)} - 6y^{(3)} + 3y'' - y' + 2y = 3\sin 2t$ as a system of first order ODEs, **then** give the same system in matrix form.

2. Consider the initial value problem $\begin{array}{ll} x_1' - x_1 - x_2 = 2e^{2t} & x_1(0) = 0 \\ x_2' - 4x_1 - x_2 = e^{2t} & x_2(0) = 1 \end{array}$.

   (a) Write the IVP in matrix form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$, $\mathbf{x}(0) = \mathbf{c}$.

   (b) Solve the associated homogeneous system $\mathbf{x}' = A\mathbf{x}$ to obtain the homogeneous solution $\mathbf{x}_h$. It will contain two arbitrary constants.

   (c) Guess a particular solution of the form $\mathbf{x}_p = \begin{bmatrix} a \\ b \end{bmatrix} e^{2t}$ and substitute it into $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$. You will be able to divide out the exponential, then arrive at a system of equations that you can solve for a and b.

   (d) The solution to the differential equation is $\mathbf{x} = \mathbf{x}_h + \mathbf{x}_p$. Write it out.

   (e) Apply the initial conditions to obtain a system of equations (not differential equations) to solve to find the constants. Find them and write out your final solution to the IVP.

3. Consider the initial value problem $y'' + 3y' + 2y = 5\cos 2t$, $y(0) = 1$, $y'(0) = -2$.

   (a) Write the ODE as a system of two first order ODEs.

   (b) Write the system in matrix form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$.

   (c) Write the initial values in terms of the new variables and then give the initial conditions in vector form.

   (d) Solve the system, **using variation of parameters to find the particular solution**.

   (e) Apply the initial conditions to obtain values for the constants. Find them and write out your final solution to the IVP.

2. $\mathbf{x}' = \frac{3}{2}\begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + \frac{1}{2}\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-t} - \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{2t}$ Any equivalent form is acceptable.

3. $\mathbf{x}' = \frac{9}{4}\begin{bmatrix} -1 \\ 2 \end{bmatrix} e^{-2t} - 8\begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{-t} + \begin{bmatrix} -\frac{1}{4}\cos 2t + \frac{3}{4}\sin 2t \\ \frac{3}{2}\cos 2t + \frac{1}{2}\sin 2t \end{bmatrix}$ Any equivalent form is acceptable.