Math 322 ASSIGNMENT 25, SPRING 2013 **Due at 3 PM Wednesday, June 5th**

1. Evaluate the integral $\int_{\tau=0}^{t} e^{\tau}\sin(t-\tau)\,d\tau$ using integration by parts. Remember that the integration by parts formula is

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du \tag{1}$$

and use the following hints if needed:

- Let $u=\sin(t-\tau)$ and $dv=e^{\tau}d\tau$ apply (1). Be careful to use the chain rule!
- Apply integration by parts again, this time letting $u=\cos(t-\tau)$ and letting $dv=e^{\tau}d\tau$ again.
- The original integral should have reappeared, with a minus sign. Add it to both sides, then multiply both sides by $\frac{1}{2}$.

2. (a) Find the Laplace transform of your answer to Exercise 1.
   (b) Get a common denominator for your answer to (a) and add all the fractions. Simplify.
   (c) Your answer to (b) is the product of two fractions. Find the inverse Laplace transform of each.

3. Solve the initial value problem from Exercise 1 of Assignment 22 using Laplace transforms, as follows.
   (a) Transform the two ODEs, using the given initial conditions. Both of your equations should contain the two transform functions $X_1(s)$ and $X_2(s)$.
   (b) Solve the first equation for $X_2(s)$. Substitute the result into the second equation and solve for $X_1(s)$. Then substitute that result back into the equation you solved for $X_2(s)$. You now have expressions for X_1 and X_2 only in terms of s.
   (c) Use Wolfram Alpha to obtain $x_1(t)$ and $x_2(t)$ from $X_1(s)$ and $X_2(s)$. Done!

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