Math 322 ASSIGNMENT 26, SPRING 2013 **Due at 3 PM Thursday, June 6th**

1. Use the convolution rule to give the Laplace transform of each of the following functions. Label each transform with its name, of course!

   (a) $f(t) = \int_0^t (t-\tau)^3 e^{-5\tau}\, d\tau$ (b) $g(t) = \int_0^t e^{2(t-\tau)} \cos(3t)\, d\tau$ (c) $h(t) = \int_0^t \sin 2\tau \, \cos 3(t-\tau)\, d\tau$

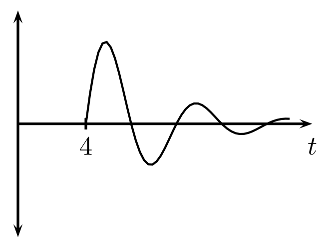
2. Give the inverse Laplace transform of each of the following as a convolution.

   (a) $F(s) = \dfrac{5s}{(s+2)(s^2+9)}$ (b) $G(s) = \dfrac{6}{(s-5)(s^2+9)}$ (c) $H(s) = \dfrac{7}{s^4(s+3)}$

3. Give the inverse Laplace transform of $F(s) = \dfrac{s\,G(s)}{s^2+5}$ as a convolution involving the function $g(t)$.

4. Find the solution to the initial value problem $y'' + 9y = g(t)$, $y(0) = -2$, $y'(0) = 1$. Your solution will contain a convolution involving the unknown function $g(t)$.

5. Here is an interesting **optional** problem: The graph of the solution to the IVP of Exercise 2 of Assignment 23 is shown to the right; recall that the forcing function for that exercise was a unit impulse at time four. Determine another impulse that can be added to the existing forcing function so that the solution goes to zero after exactly one complete cycle. Check your answer by solving the new IVP and looking at the graph given by Wolfram Alpha when you take the inverse transform. **A special prize will be awarded to anyone who can solve this by Friday and give an explanation (to the class) of how their solution was obtained.**



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