Math 322 ASSIGNMENT 4, SPRING 2013 **Due at 3 PM Wednesday, April 10th**

1. In this exercise you will use matrix methods to solve the system $\begin{array}{rcl} x_1' & = & x_1 + 2x_2 \\ x_2' & = & 3x_1 + 2x_2 \end{array}$, where x_1 and x_2 are both functions of t.

   (a) Write the system in matrix form. Remember this is the form $\mathbf{x}' = A\mathbf{x}$, but give all the elements/components of $\mathbf{x}'$, A and $\mathbf{x}$.

   (b) Give the general form of the guess $\mathbf{x}$ for the solution, and give its derivative. Substitute it into $\mathbf{x}' = A\mathbf{x}$ to get the eigenvalue/eigenvector equation $A\mathbf{c} = \lambda\mathbf{c}$.

   (c) Use the one of the eigenvalue/eigenvector calculators at the class web page to find the $(\lambda_i, \mathbf{c}_i)$ pairs for the coefficient matrix A.

   (d) Write the solution to the system in vector form $\mathbf{x} = \mathbf{c}_1 e^{\lambda_1 t} + \mathbf{c}_2 e^{\lambda_2 t}$.

   (e) Write the solution to the system as two separate functions and **put a box around them**. Then verify your solution by giving x_1', $x_1 + 2x_2$, x_2' and $3x_1 + 2x_2$. If the appropriate ones of these aren't equal, find your error!

2. Demonstrate how to find the eigenvalues and eigenvectors for the coefficient matrix A from the previous exercise **by hand**.

3. For each of the following compute the inverse Laplace transform, using the suggestions provided. **Show the suggested algebraic manipulations and check your answers using the Wolfram® inverse Laplace transform calculator.** The link to it can be found at the class web page. *I will grade these based only on the effort you've shown, and I'll not look carefully at your computations unless you put a note (like a big question mark) next to any that you can't obtain the correct answer to.*

   (a) Find the inverse Laplace transform of $F(s) = \dfrac{2}{5s+1}$ by first pulling the 2 off the top, factoring 5 out of the bottom and pulling it out also. Then find the appropriate form in the table.

   (b) Find the inverse Laplace transform of $F(s) = \dfrac{5}{s^2+4}$. Again, begin by pulling the 5 off the top. You will then need a value on top to match the form of the transform of the sine function. Put that number on top, while at the same time multiplying by its reciprocal "out front." Then apply the table.

   (c) Find the inverse Laplace transform of $F(s) = \dfrac{2s-1}{s^2+9}$ by first applying the (underrated and often very useful) fact that $\dfrac{a \pm b}{c} = \dfrac{a}{c} \pm \dfrac{b}{c}$. Then apply the techniques of the previous two exercises.

   (d) Use partial fractions to find the inverse transform of $F(s) = \dfrac{1}{s^2+3s-10}$.

   (e) Use partial fractions to find the inverse transform of $F(s) = \dfrac{2s+1}{s^2+3s-10}$.

   (f) Find the inverse Laplace transform of $F(s) = \dfrac{s}{(s-2)^2+9}$ as follows: First subtract and add 2 (in that order!) to the numerator, which leaves its value the same. The use the "splitting method" of part (c), keeping $s-3$ together in the first part. You can then use the last formula on the table for the first part, and you'll be able to apply the next to last part after using the method of part (b). *When checking with Wolfram®, be sure to look below at the alternate forms.*

   (g) Use a masterful combination of all the techniques you have used so far to find the inverse Laplace transform of $F(s) = \dfrac{7s-2}{(s+3)^2+5}$! Of course you will need to use the facts that $s+3 = s-(-3)$ and $5 = (\sqrt{5})^2$...