Math 322 ASSIGNMENT 5, SPRING 2013 **Due at 3 PM Friday, April 12th**

1. In this exercise you'll use the Laplace transform to solve the initial value problem

$$y'' + 4y' + 3y = 5\sin 2t, \qquad y(0) = 2, \qquad y'(0) = -1$$

You will use technology to do it - I tried the algebra by hand and it was pretty tedious and annoying!

(a) Take the Laplace transform of both sides of the equation.

(b) Solve for $Y(s)$, leaving the right hand side as two separate fractions.

(c) Use Wolfram® to calculate the inverse Laplace transform of each of the two fractions, and give the resulting solution.

(d) Combine like terms to get your final solution.

2. Consider the system $\begin{aligned} x_1' &= 4x_1 - 2x_2 \\ x_2' &= 5x_1 - 7x_2 \end{aligned}$ of differential equations.

(a) Give the matrix form of the system.

(b) Find the eigenvalues and eigenvectors of the coefficient matrix by hand, showing clearly how you do it. Then check them with your calculator or an online tool (or your neighbor).

(c) Give the solution to the system **in vector form**.

(d) Suppose that $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, so we now have an initial value problem. Substitute this into your solution, and write the system of two equations in two unknowns that results. Then solve that system **using the addition method**, showing how you do it.

(e) Give your final solution to the IVP as the two separate functions x_1 and x_2.