Math 322 ASSIGNMENT 6, SPRING 2013 **Due at 3 PM Tuesday, April 16th**

1. Find the Laplace transform of $f(t) = \cos \omega t$ from the definition and by applying the formula $\int e^{at} \cos bt\, dt = \dfrac{e^{at}}{a^2+b^2}(a \cos bt + b \sin bt)$. **Show your work, integrating from zero to** R **and then applying a limit to the result.**

2. Find the Laplace transform of $f(t) = \sin \omega t$ by applying the identity $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ and using the linearity of the transform and the already known transform of e^{at}. Remember that i is just a constant, and can be treated as such. You will end up with two fractions; get a common denominator and combine them.

3. (a) Give the values of i^2 through i^6 in terms of i or 1.

   (b) Give the first eight terms of the power series for $e^{i\theta}$, then modify it as follows:
   - Use your answers to (a) to eliminate powers of i.
   - Group the terms without i, and those with i.
   - Factor out the i from the terms having it.
   - You should now have two series, one without i and one with it. Replace each with the functions they represent.

4. Consider the function $f(t) = \begin{cases} t^2 & \text{if } 0 \le t < 1 \\ 2-t & \text{if } 1 \le t < 2 \\ 0 & \text{if } t \ge 2 \end{cases}$

   (a) Sketch a *neat* graph of the function.

   (b) Write the function as a single function using the unit step function $u(t)$.

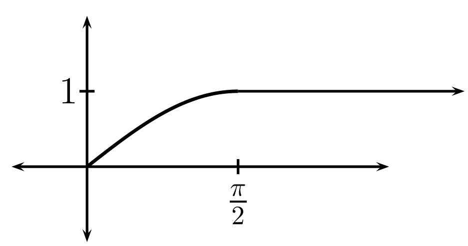
5. *Neatly* sketch the graph of each of the following:

   (a) $f(t) = t - tu(t-2)$ (b) $g(t) = t + (2-t)u(t-2)$

   (c) $h(t) = t[1 - u(t-2)] + (4-t)[u(t-2) - u(t-4)]$

6. Give the Laplace transforms of each of the functions in the previous exercise. Simplify your answers when possible.

7. (a) Use the unit step function $u(t)$ to write the function $f(t)$ whose graph is shown to the right as a single function. The initial part of the graph is the sine function, then the constant function one.



   (b) Give the Laplace transform of the function.