Math 322 Assignment 7, Spring 2013 **Due at 3 PM Thursday, April 18th**

1. Consider the function $f(t) = \begin{cases} 0 & \text{if} \quad t < 0 \\ 2 & \text{if} \quad 0 \le t < 1 \\ 1 & \text{if} \quad 1 \le t < 3 \\ 5 & \text{if} \quad t \ge 3 \end{cases}$

   (a) Sketch a graph of the function.

   (b) Give a single equation of the function, using step functions.

   (c) Find the Laplace transform of the function.

2. Let $g(t)$ be the function that is zero until time one and then ramps up (linearly) to a value of six at time $t = 5$. Repeat the parts of Exercise 1 for this function.

3. The point of this exercise is to change the expression $4t - t^2$ into a function of $(t - 4)$.

   (a) Replace t everywhere with $[(t - 4) + 4]$. Perform the operations of multiplying by four and squaring *without breaking up or "foiling out" the* $(t - 4)$. See the notes or the video on Laplace transforms with step functions for how the squaring part goes.

   (b) Combine any like terms, still not breaking up or "foiling out" the $(t - 4)$. Your final result should be something of the form $a(t - 4) - (t - 4)^2$ for some constant a.

   (c) Your result from (b) is $f(t - 4)$ for some function f. Give the function.

4. (a) Sketch the graph of the function $h(t) = (4t - t^2)[u(t) - u(t - 4)]$.

   (b) Find the Laplace transform of the function. You will need to distribute the $4t - t^2$ to both step functions. The first part will be ready to go, and the result of the previous exercise should be useful for the second part.

5. Find the inverse Laplace transform of each of the following using $\mathscr{L}^{-1}[e^{-cs}F(s)] = f(t-c)u(t-c)$.

   (a) $G(s) = \dfrac{3e^{-2s}}{s^2}$

   (b) $G(s) = \dfrac{7e^{-5s}}{s - 2}$

   (c) $G(s) = \dfrac{2se^{-3s}}{s^2 + 25}$

   (d) $G(s) = e^{-s}\left(\dfrac{1}{s^3} - \dfrac{1}{s^2} + \dfrac{1}{s}\right)$