Math 322 ASSIGNMENT 9, SPRING 2013 **Due at 3 PM Wednesday, April 24th**

1. Consider the matrix $\begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}$.

   (a) Find the eigenvalues. The characteristic equation can't be (easily) factored, so use the quadratic formula. You should get complex conjugate eigenvalues $a \pm bi$.

   (b) Find the system of equations used to find the eigenvector for the eigenvalue with the form $a + bi$. Previously we have obtained two equations that are scalar multiples of each other, but it is not apparent that they are in this case. Try multiplying the first equation by $a + bi$ and see what you get.

   (c) Solve the first equation for the second unknown, in terms of the first. Let the first unknown be one to obtain the eigenvector.

   (d) Find the second eigenvector in a similar manner. What do you notice about the eigenvectors?

2. In this exercise you'll solve the IVP $\mathbf{x}' = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

   (a) Using your results from Exercise 1, solve the system of equations $\mathbf{x}' = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}\mathbf{x}$ in exactly the manner that you would do it if the eigenvalues and eigenvectors were real.

   (b) Substitute The initial values to get a system of two equations in two unknowns. I think this is easiest by solving the equation with no complex numbers for one of the unknowns and substituting the result into the second equation. At the end you will need to multiply by $\frac{i}{i}$ to "rationalize" the denominator. **Both constants will be complex.**

   (c) Write your answer as two separate equations, one for x_1 and one for x_2. Apply Euler's formula (remember that $e^{(a+bi)t} = e^{at}e^{bit}$) and combine like terms.

3. In this exercise you will solve the IVP $y'' + 4y = h(t), \quad y(0) = 0, \quad y'(0) = 0$, where $h(t)$ is the function from Exercise 2 of Assignment 7, that has value zero until time one, then "ramps up" (linearly) to value six at five seconds, then stays six from then on. Recall that that function can be expressed using step functions as

$$h(t) = \tfrac{3}{2}(t-1)u(t-1) - \tfrac{3}{2}(t-5)u(t-5).$$

   (a) Take the Laplace transform of both sides of the ODE and solve for $Y(s)$.

   (b) Use Wolfram Alpha to determine $y(t)$. Sketch a reasonably neat and large version of the second graph of the solution, but starting only at zero.

4. Solve the system $\mathbf{x}' = \begin{bmatrix} 5 & -4 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & 5 \end{bmatrix}\mathbf{x}$, using Wolfram® to get the eigenvalues and eigenvectors. Here are a couple of comments about it:

   - The zero vector is not allowed to be an eigenvector, but it *IS* possible to have an eigenvalue of zero, which you do in this case.
   - The other eigenvalue is repeated, but with only one eigenvector. You will need to find a generalized eigenvector.