② series solutions

③ systems of ODEs

$\boxed{\text{① Laplace transforms}}$

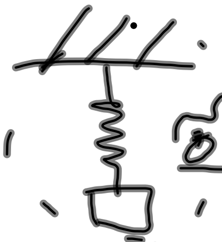
→ find $y(t)$

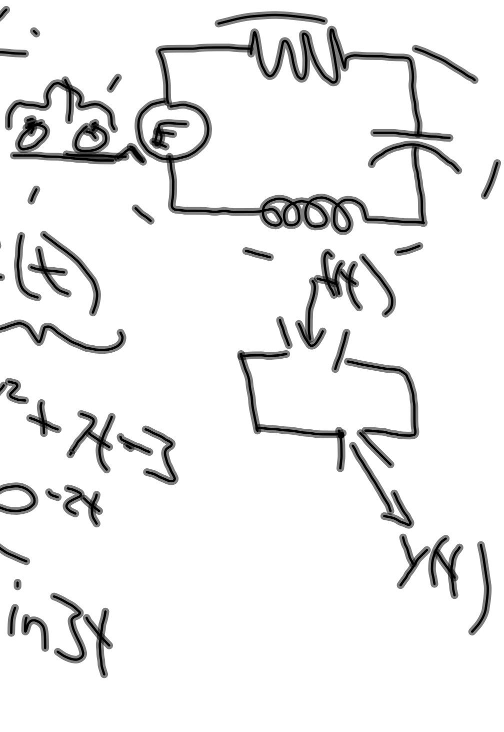
→ solve IVP

→ Why?

$\frac{d^2q}{dt^2}$

$$\underbrace{ay'' + by' + cy}_{\text{system}} = f(t)$$





$2t^2 + 7t - 3$

e^{-2t}

$\sin 3t$

$$f(t) \longrightarrow \boxed{\mathcal{L}} \longrightarrow F(s)$$

What's in here?

Laplace transform

$$\int_0^\infty \underbrace{f(t)}_{\text{Function of } t}\,\underbrace{e^{-st}}_{\text{function of } s \,\&\, t}\,dt$$

$$\int_a^b f(t)\,\underbrace{k(t,s)}_{\text{kernel}}\,dt$$

Integral transforms

f(t)	F(s)

$$\int_1^\infty \frac{1}{x^2}\,dx \stackrel{\text{def}}{=} \lim_{R\to\infty} \int_1^R \frac{1}{x^2}\,dx$$

How to "do" $\int_0^\infty \frac{1}{x^2}\,dx$

$$\int_1^R \frac{1}{x^2}\,dx = \int_1^R x^{-2}\,dx = -x^{-1}\Big|_1^R = -\frac{1}{x}\Big|_1^R = -\frac{1}{R}+1$$

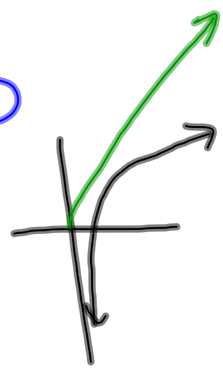
$$\int_1^\infty \frac{1}{x^2}\,dx = \lim_{R\to\infty}\int_1^R \frac{1}{x^2}\,dx = \lim_{R\to\infty}\left(-\frac{1}{R}+1\right) = 1$$

Evaluate $\int_0^\infty e^{-5t}\,dt$

$$\int_0^R e^{-5t}\,dt = -\frac{1}{5}e^{-5t}\Big|_0^R = -\frac{1}{5}e^{-5R} + \frac{1}{5}e^0 = -\frac{1}{5}e^{-5R} + \frac{1}{5}$$

$$\int_0^\infty e^{-5t}\,dt = \lim_{R\to\infty}\int_0^R e^{-5t}\,dt = \lim_{R\to\infty}\left(-\frac{1}{5}e^{-5R}+\frac{1}{5}\right) = \frac{1}{5}$$

$$\underbrace{\int_1^\infty \frac{1}{x}\,dx}_{\text{DNE}} = \ln x\Big|_1^\infty = \ln\infty - \cancel{\ln 1}^{\,0}$$



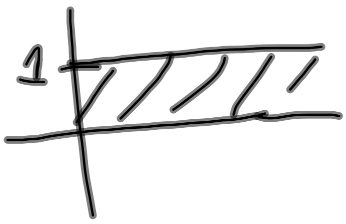
$$\mathcal{L}[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt$$

$f(t) = 1$ (at least for $t \geq 0$)

$$F(s) = \int_0^\infty e^{-st}\, dt$$

$s=0$, $s>0$, $s<0$

$s=0$: $\int_0^\infty 1\, dt$



$\mathcal{L}[f(t)]$ DNE if $s=0$

$s \neq 0$: $\int_0^\infty e^{-st}\, dt = -\frac{1}{s} e^{-st} \Big|_0^\infty = -\frac{1}{s} e^{-s\infty} + \frac{1}{s} = \frac{1}{s}$ if $s>0$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$$f(t) = t^n \qquad u = t^n \quad dv = e^{-st}dt$$

$$du = nt^{n-1} \quad v = -\frac{1}{s}e^{-st}$$

$$F(s) = \int_0^\infty t^n e^{-st}\,dt$$

$$= \cancelto{0}{-\frac{1}{s}t^n e^{-st}\Big|_0^\infty} - \int_0^\infty nt^{n-1}\left(-\frac{1}{s}e^{-st}\right)dt$$

$$= +\frac{n}{s}\int_0^\infty t^{n-1}e^{-st}\,dt$$

$$= \frac{n}{s}\mathcal{L}\left[t^{n-1}\right]$$

$$= \frac{n}{s}\,\frac{n-1}{s}\,\mathcal{L}\left[t^{n-2}\right]$$

$$= \frac{n}{s}\cdot\frac{n-1}{s}\cdots\frac{2}{s}\cdot\frac{1}{s}\mathcal{L}\left[1\right]$$

$$\mathcal{L}\left[t^n\right] = \frac{n}{s}\cdot\frac{n-1}{s}\cdot\ \cdots\ \frac{2}{s}\cdot\frac{1}{s}\cdot\frac{1}{s} = \frac{n!}{s^{n+1}}$$