Remember: $\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,dt$

Use to find $\mathcal{L}[e^{at}]$.

$$\int_0^\infty e^{at}e^{-st}\,dt = \int_0^\infty e^{(a-s)t}\,dt$$

$$\int_0^\infty e^{-(s-a)t}$$

$$= \frac{1}{a-s}e^{(a-s)t}\Big|_0^\infty$$

$\to 0$ if $a-s<0$

$a<s$

$$= \frac{1}{a-s}e^{(a-s)\infty} - \frac{1}{a-s}$$

$$= \frac{1}{s-a} \quad s>a$$

$$\int e^{at}\sin bt\,dt = \frac{e^{at}}{a^2+b^2}(a\sin bt - b\cos bt)$$

$$\mathcal{L}[\sin\omega t] = \int_0^\infty e^{-st}\sin\omega t\,dt$$

$$= \left.\frac{e^{-st}}{(-s)^2+\omega^2}(-s\sin\omega t - \omega\cos\omega t)\right|_0^\infty$$

0 when $t=\infty$

$$= \qquad - \frac{1}{s^2+\omega^2}(-\omega)$$

$$= \frac{\omega}{s^2+\omega^2}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta)$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

add

$$e^{i\theta} + e^{-i\theta} = 2\cos\theta$$

$$\boxed{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$

subtract

$$e^{i\theta} - e^{-i\theta} = 2i\sin\theta$$

$$\boxed{\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}}$$

Even

$f(-x) = f(x)$

Odd

$f(-x) = -f(x)$

$$\mathcal{L}[\cos\omega t] = \mathcal{L}\left[\frac{e^{i\omega t}+e^{-i\omega t}}{2}\right]$$

$$= \frac{1}{2}\mathcal{L}\left[e^{i\omega t}+e^{-i\omega t}\right]$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a}$$

$$= \frac{1}{2}\left\{\mathcal{L}[e^{i\omega t}]+\mathcal{L}[e^{-i\omega t}]\right\}$$

$$= \frac{1}{2}\left[\frac{1}{s-i\omega}+\frac{1}{s+i\omega}\right]$$

$$(i\omega)^2 = (i\omega)(i\omega) = -\omega^2$$

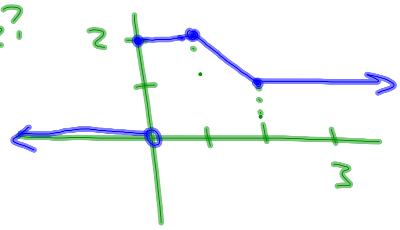
$$= \frac{1}{2}\left[\frac{(s+i\omega)+(s-i\omega)}{(s+i\omega)(s-i\omega)}\right]$$

$$= \frac{1}{\cancel{2}}\,\frac{\cancel{2}s}{s^2-\underbrace{(i\omega)^2}_{=-\omega^2}} = \frac{s}{s^2+\omega^2}$$

$ay''+by'+cy=f(t)$

$$f(t)=\begin{cases} 0 & \text{for } t<0 \\ 2 & \text{for } 0\le t<1 \\ 3-t & \text{for } 1\le t<2 \\ 1 & \text{for } t\ge 2 \end{cases} \qquad y=-t+3$$

Picture?

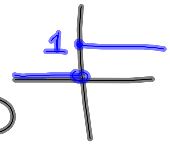


$F(s)=?$

Goal

$f(t)= \ldots\ldots$

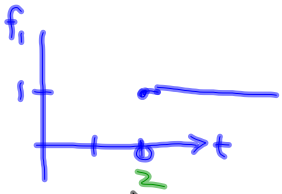
Use $u(t)=\begin{cases} 0 & \text{for } t<0 \\ 1 & \text{for } t\ge 0 \end{cases}$



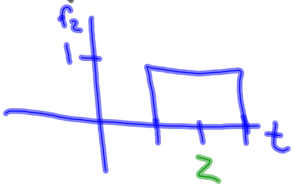
unit step function

Graphs of

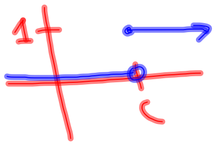
$f_1(t)=u(t-2)$



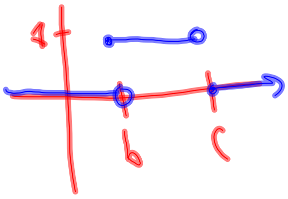
$f_2(t)=u(t-1)-u(t-3)$



$u(t-c)$



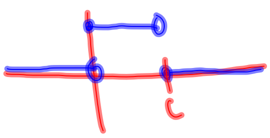
$u(t-b)-u(t-c)$

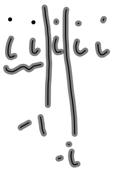


$u(t)-u(t-c)$

or

$1-u(t-c)$





$(i\theta)^5=i^5\theta^5$