Eigenvector

$$\begin{bmatrix} 10 & -2 \\ 5 & -1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$5c_1 - c_2 = 0$$

$$5c_1 = c_2$$

$$c_1 = \frac{1}{5} c_2$$

$$\begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

Apr 17-9:01 AM

Assignment 5. #1

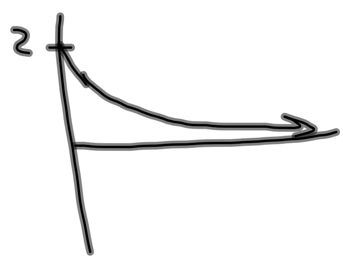
$$y'' + 4y' + 3y = \underbrace{5\sin 2t}_{\text{forcing}} \qquad y(0) = 2, \quad y'(0) = -1$$

$r^2 + 4r + 3 = 0$

$(r+1)(r+3) = 0$

$r = -1, -3$

$y_h = c_1 e^{-t} + c_2 e^{-3t}$



$$Y(s)\left[\underbrace{s^2+4s+3}_{\text{system}}\right]\underbrace{-2s-7}_{\text{ICs}} = \underbrace{\frac{10}{s^2+4}}_{\text{forcing}}$$

$$Y(s) = \frac{2s+7}{s^2+4s+3} + \frac{10}{(s^2+4)(s^2+4s+3)}$$

$$y(t) = \underbrace{\boxed{e^{-t}, e^{-3t}}}_{\text{response to ICs}} + \underbrace{\boxed{e^{-t}, e^{-3t}, \sin 2t, \cos 2t}}_{\text{response to } f(t)}$$

$$\mathcal{L}[f(t)u(t-c)] = \int_0^\infty f(t)\underline{\underline{u(t-c)}}e^{-st}\,dt$$

$$= \int_c^\infty f(t)e^{-st}\,dt = ?$$

$$\mathcal{L}[f(t-c)u(t-c)] = \int_0^\infty f(t-c)u(t-c)e^{-st}\,dt$$

Let $u = t-c$

$du = dt$

$t = u+c$

When $t=c$, $u=0$

When $t=\infty$, $u=\infty$

$$= \int_{t=c}^\infty f(t-c)e^{-st}\,dt$$

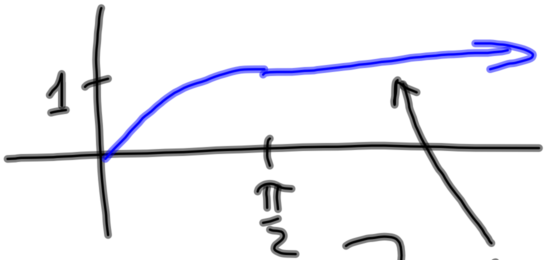
$$= \int_{u=0}^\infty f(u)\underbrace{e^{-s(u+c)}}_{e^{-su}\underline{\underline{e^{-cs}}}}\,du$$

$$= e^{-cs}\underbrace{\int_0^\infty f(u)e^{-su}\,du}$$

$$= e^{-cs}F(s)$$

Apr 17-9:14 AM

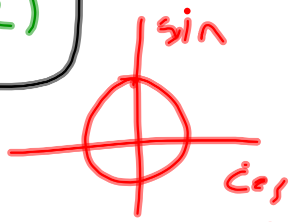
Assn 6, #7



$$f(t) = \sin t\left[u(t-0) - u\left(t-\tfrac{\pi}{2}\right)\right] + 1u\left(t-\tfrac{\pi}{2}\right)$$

$$f(t) = \sin t\, u(t) + \left[1-\sin(t)\right]u\left(t-\tfrac{\pi}{2}\right)$$

$$1u\left(t-\tfrac{\pi}{2}\right) - \boxed{\sin(t)\,u\left(t-\tfrac{\pi}{2}\right)}$$



$$\sin\left[\left(t-\tfrac{\pi}{2}\right)+\tfrac{\pi}{2}\right] = \cos\left(t-\tfrac{\pi}{2}\right)\cancelto{1}{\sin\tfrac{\pi}{2}} + \sin\left(t-\tfrac{\pi}{2}\right)\cancelto{0}{\cos\tfrac{\pi}{2}}$$

$\sin(a+b) = ?$

$$= \cos\left(t-\tfrac{\pi}{2}\right) \qquad \cos\left(t-\tfrac{\pi}{2}\right)u\left(t-\tfrac{\pi}{2}\right)$$

$$e^{(a+b)i} = \underline{\cos(a+b)} + i\underline{\sin(a+b)}$$

$$\|$$

$$e^{ai}e^{bi} = (\cos a + i\sin a)(\cos b + i\sin b)$$

$$= \underline{(\cos a\cos b - \sin a\sin b)} + i\,\underline{(\cos a\sin b + \sin a\cos b)}$$

$$\mathcal{L}\left[\cos\left(t-\tfrac{\pi}{2}\right)u\left(t-\tfrac{\pi}{2}\right)\right] = \cancel{?}\quad e^{-\frac{\pi}{2}s}\cdot\frac{s}{s^2+1}$$

$$\mathcal{L}\left[f(t-c)u(t-c)\right] = e^{-cs}F(s)$$

$c=\frac{\pi}{2}$ $\quad f(t-c)=\cos 1\left(t-\frac{\pi}{2}\right)$

$$f(t)=\cos 1t \Longrightarrow F(s)=\frac{s}{s^2+1}$$

$$\cos\left(2t-\tfrac{\pi}{2}\right)=\cos 2\left(t-\tfrac{\pi}{4}\right)$$

$$\mathcal{L}\left[\cancel{1}\,u(t-c)\right] = e^{-cs}\mathcal{L}[1]$$

$$= \frac{e^{-cs}}{s}$$

$f(t-c)$

$f(t)=1$

$5u(t-3)$

$$\mathcal{L}\left[f(t-c)u(t-c)\right] = e^{-cs}F(s)$$

$$\mathcal{L}^{-1}\left[e^{-cs}F(s)\right] = f(t-c)u(t-c)$$

$$\mathcal{L}^{-1}\left[\frac{e^{-5s}}{s+2}\right] = \mathcal{L}^{-1}\left[e^{-5s}\underbrace{\frac{1}{s-(-2)}}\right]$$

$c=5$ $\quad F(s) \to f(t)=e^{-2t}$

$$= e^{-2(t-5)}u(t-5)$$

$$= e^{10}e^{-2t}u(t-5)$$