① Solve $\vec{x}' = \underbrace{\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}}_{A} \vec{x}$, $\vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underbrace{\det(A - \lambda I)}= 0$ $\quad A - \lambda I = \begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}$

$\lambda^2 - 4\lambda + 3 = 0$ $\quad$ Solve $(A - \lambda I)\vec{c} = \vec{0}$

$\lambda = 3, 1$

$\lambda = 3: \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \vec{c} = \vec{0}$

$c_1 + c_2 = 0$

$c_1 = -c_2$ $\quad \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$$\boxed{\vec{x} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} e^{3t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}}$$

Sol to system

How about the IVP?

$\Rightarrow$ Apply $\vec{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$-c_1 + c_2 = 1$

$c_1 + c_2 = 1$

$2c_2 = 2$

$c_2 = 1$

$c_1 = 0$

$$\boxed{\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t}}$$

Vector form

$x_1 = e^t$

$x_2 = e^t$

② Solve $\vec{x}' = \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix} \vec{x}$

$$\det(A-\lambda I) = \det\begin{bmatrix} 3-\lambda & -18 \\ 2 & -9-\lambda \end{bmatrix}$$

$$= (3-\lambda)(-9-\lambda) - (-36)$$

$$= -27 + 6\lambda + \lambda^2 + 36$$

$$= \lambda^2 + 6\lambda + 9 = (\lambda+3)(\lambda+3)$$

$$= (\lambda+3)^2$$

$\lambda = -3$ is an eigenvalue with algebraic multiplicity 2.

$(A-\lambda I)\vec{c} = \vec{0}$ Solve to find eigenvector(s)

$$\begin{bmatrix} 6 & -18 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0 \qquad c_1 - 3c_2 = 0 \qquad c_1 = 3c_2 \qquad \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

Only 1 eigenvector is stated by saying $\lambda = -3$ has geometry multiplicity one.

$$\vec{x} = c_1\begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{-3t} + c_2\begin{bmatrix} \quad \end{bmatrix} e^{-3t}$$

?

$$\vec{x}' = \begin{bmatrix} 3 & -18 \\ 2 & -9 \end{bmatrix} \vec{x} \qquad \lambda = -3, \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\vec{x} = d_1 \begin{bmatrix} \\ \end{bmatrix} e^{\lambda_1 t} + d_2 \begin{bmatrix} \\ \end{bmatrix} t e^{\lambda_1 t}$$

$$\vec{x} = c_{1,1} \vec{k}_1 e^{\lambda_1 t} + c_{1,2} \left[\vec{k}_1 t + \vec{p}_1 \right] e^{\lambda_1 t}$$

$$\vec{x} = c_{1,1} \begin{bmatrix} 3 \\ 1 \end{bmatrix} e^{-3t} + c_{1,2} \left[\begin{bmatrix} 3 \\ 1 \end{bmatrix} t + \begin{bmatrix} \\ \end{bmatrix} \right] e^{-3t}$$

$$(A + 3I)\vec{p} = \vec{k}$$

$$\begin{bmatrix} 6 & -18 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad \begin{matrix} 6p_1 - 18p_2 = 3 \Rightarrow p_1 - 3p_2 = \frac{1}{2} \\ 2p_1 - 6p_2 = 1 \Rightarrow p_1 - 3p_2 = \frac{1}{2} \end{matrix}$$

$$p_1 - 3p_2 = \frac{1}{2} \qquad p_2 = 0 \Rightarrow \frac{1}{2} \qquad \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}$$

our choice

Apr 19-9:38 AM

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

2×2

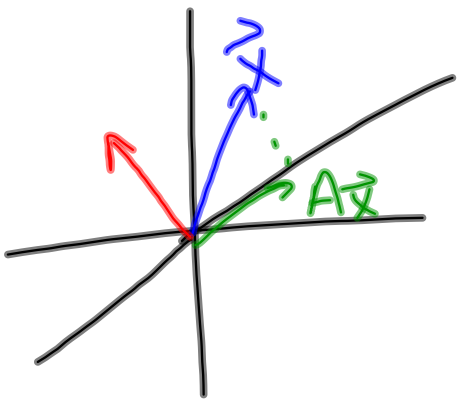
$n = 2$

$$\lambda_1 = 3, \lambda_2 = 1$$

$$\downarrow \qquad \downarrow$$

$$\vec{k}_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \vec{k}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\vec{X} = c_1 k_1 e^{\lambda_1 t} + c_2 k_2 e^{\lambda_2 t}$$



$$A\vec{x}=\lambda\vec{x}$$