Find the Laplace transform of $\frac{3}{4}(t-2)u(t-5)$

$$\mathcal{L}[f(t-c)u(t-c)] = e^{-cs}F(s)$$

$$f(t) = \frac{3}{4}[(t-5)+3]u(t-5)$$

$$\underbrace{\frac{3}{4}(t-5)}u(t-5)+\frac{9}{4}u(t-5)$$

$$f(t-c) = \frac{3}{4}(t-5)$$

$$f(t) = \frac{3}{4}t$$

$$F(s) = \frac{3}{4}\cdot\frac{1}{s^2} = \frac{3}{4s^2}$$

$$F(s) = e^{-5s}\cdot\frac{3}{4s^2} + \frac{9}{4}\frac{e^{-5s}}{s}$$

$$= \frac{3e^{-5s}}{4s^2} + \frac{9e^{-5s}}{4s}$$

Apr 22-8:51 AM

Office Mon 10-11, 12-1, 2-3

Tues 10-3:30

other if you want

(5) a) $g(t) = 3(t-2)u(t-2)$

b) $g(t) = 7e^{2(t-5)}u(t-5)$

c) $g(t) = 2\cos 5(t-3)u(t-3)$

d) $g(t) = \left[\frac{1}{2}(t-1)^2 - (t-1) + 1\right]u(t-1)$

Assn 8 #3

$\lambda = 4, 1 \rightarrow$ multiplicity 2 $\quad (A-\lambda I)\vec{p} = \vec{k}$

$k_1 \quad k_2$

$$\vec{x} = c_1 \begin{bmatrix} \\ \\ \\ \end{bmatrix} e^{0} + c_2 \begin{bmatrix} \\ \\ \\ \end{bmatrix} e^{0} + c_3 \begin{bmatrix} \\ \\ \\ \end{bmatrix} e^{0}$$

$\lambda = 1$

$$A - \lambda I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

$$(A - \lambda I)\vec{x} = \vec{0} \qquad x_1 + x_2 + x_3 = 0$$

x_3 is free

$x_3 = t$

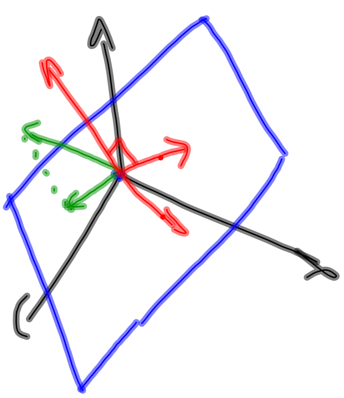
$x_2 = s$

$x_1 = -s - t$

$$\begin{bmatrix} 1 & 0 & a & 0 \\ 0 & 1 & b & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -s-t \\ s \\ t \end{bmatrix} = \begin{bmatrix} -s \\ s \\ 0 \end{bmatrix} + \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix}$$

$$= s \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$



Apr 22-9:16 AM

$y'' + 6y' + 9y = 0$

$r^2 + 6r + 9 = 0$

$(r+3)(r+3) = 0$

$r = -3 \qquad y = c_1 e^{-3t} + \cancel{c_2 e^{-3t}}$

Why te^{-3t}? $\qquad y = Ce^{-3t}$

How did someone figure it out?

What if $y = u(t)e^{-3t}$?

$y' = -3ue^{-3t} + u'e^{-3t}$

$y'' = 9ue^{-3t} - 3u'e^{-3t} - 3u'e^{-3t} + u''e^{-3t}$

$= 9ue^{-3t} - 6u'e^{-3t} + u''e^{-3t}$

$0 = \cancel{9ue^{-3t}} \cancel{-6u'e^{-3t}} + u''e^{-3t} \cancel{-18ue^{-3t}} \cancel{+6u'e^{-3t}} \cancel{+9ue^{-3t}}$

$0 = u''\cancel{e^{-3t}}$ (Normally)

$au'' + bu' = 0$ Let $u' = v$

$av' + bv = 0$

$u'' = 0$

$u' = c_1$

$u = c_1 t + c_2 \qquad y = c_1 e^{-3t} + c_2 te^{-3t}$

$y = ue^{-3t} = (c_1 t + c_2)e^{-3t}$

Apr 22-9:29 AM

$\vec{x}' = A\vec{x}$ Only one eigenvalue λ_1 with
$\hookrightarrow 2\times 2$ only one eigenvector $\vec{k}$

One part of solution is $c_1\vec{k}e^{\lambda t}$

Guess other part looks like $c_1\underbrace{\vec{k}te^{\lambda t}}_{\uparrow} + c_2\vec{p}e^{\lambda t}$

$$\vec{x}' = \lambda c_1\vec{k}te^{\lambda t} + c_1\vec{k}e^{\lambda t} + \lambda c_2\vec{p}e^{\lambda t}$$

$$= A\vec{x} = A(c_1\vec{k}te^{\lambda t}) + A(c_2\vec{p}e^{\lambda t})$$

$$= c_1te^{\lambda t}A\vec{k} + c_2e^{\lambda t}A\vec{p}$$

$$\lambda c_1\vec{k}t + c_1\vec{k} + \lambda c_2\vec{p} = c_1tA\vec{k} + c_2A\vec{p}$$

$$\cancel{c_1}A\vec{k} = \lambda\cancel{c_1}\vec{k}$$

$$A\vec{k} = \lambda\vec{k}$$

$$c_1\vec{k} + \lambda c_2\vec{p} = Ac_2\vec{p}$$

$$c_1\vec{k} = (A - \lambda I)c_2\vec{p}$$

$$(A - \lambda I)\vec{p} = \vec{k}$$

generalized eigenvector

true eigenvector

Apr 22-9:38 AM