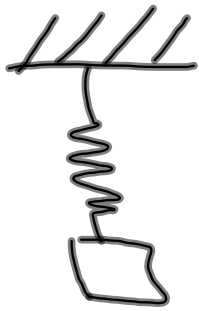
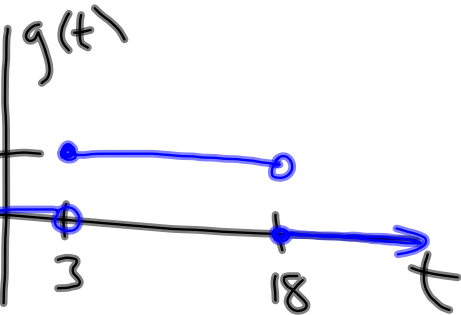


$$2y'' + 2y' + 5y = \cancel{g(t)} 2u(t-3) - 2u(t-18)$$



$$2y'' + 2y' + 5y = 0$$

$$2r^2 + 2r + 5 = 0$$

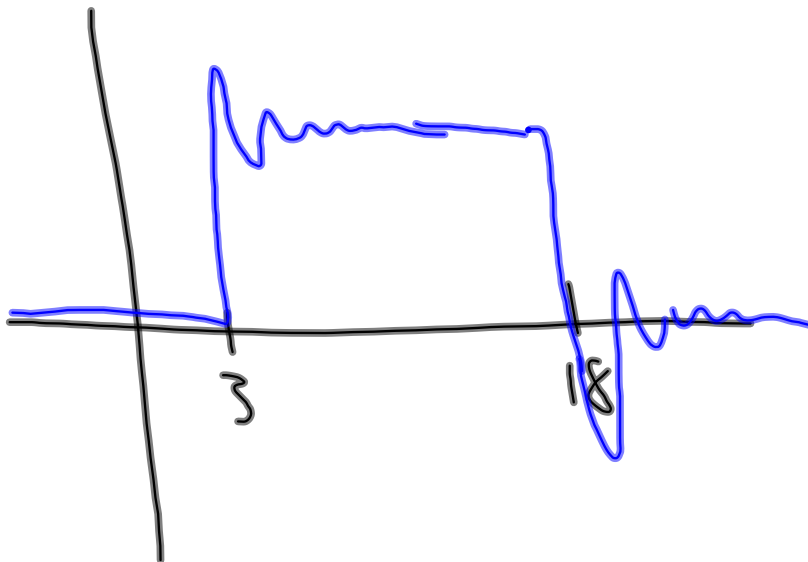


$$r = \frac{-2 \pm \sqrt{4 - 40}}{4}$$

$$r = -\frac{1}{2} \pm \frac{3}{2}i$$

$$y = c_1 e^{\left(\frac{1}{2} + \frac{3}{2}i\right)t} + c_2 e^{\left(\frac{1}{2} - \frac{3}{2}i\right)t}$$

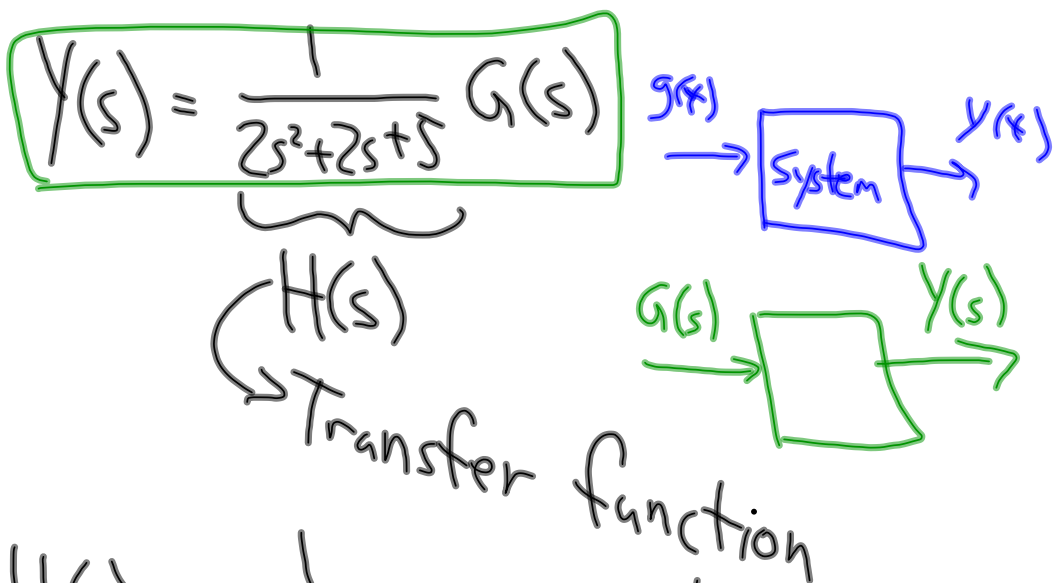
$$= e^{-\frac{1}{2}t} \left( d_1 \cos \frac{3}{2}t + d_2 \sin \frac{3}{2}t \right)$$



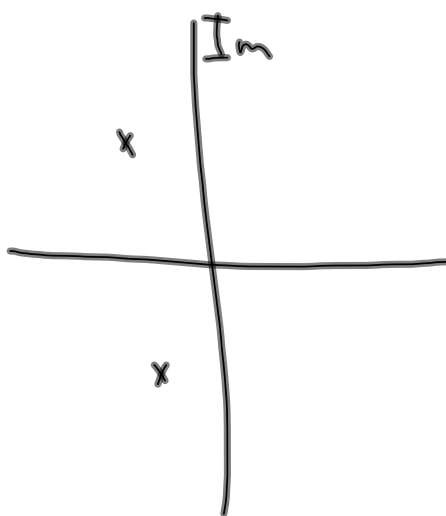
$$2y'' + 2y' + 5y = g(t) \quad \begin{matrix} y(0) = 0 \\ y'(0) = 0 \end{matrix}$$

$$2s^2Y(s) + 2sY(s) + 5Y(s) = G(s)$$

$$Y(s)(2s^2 + 2s + 5) = G(s)$$



$$H(s) = \frac{1}{2s^2 + 2s + 5} = \frac{1}{(s + \frac{1}{2} + \frac{\sqrt{3}}{2}i)(s + \frac{1}{2} - \frac{\sqrt{3}}{2}i)}$$



Poles

$$s = \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right), \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$$

$$\vec{x}' = A\vec{x} \quad (\lambda-3)^2(\lambda+2)^4(\lambda-1) = 0$$

$$(\lambda-3)(2\lambda^2+2\lambda+5) = 0$$

$$\vec{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x} \quad \text{"normally"}$$

①  $\lambda_1, \lambda_2, \vec{k}_1, \vec{k}_2$

$$\vec{x} = c_1 \vec{k}_1 e^{\lambda_1 t} + c_2 \vec{k}_2 e^{\lambda_2 t}$$

All holds if  $\lambda_1, \lambda_2, \vec{k}_1, \vec{k}_2$  are complex.

②  $\vec{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x} \quad \lambda, \vec{k}_1, \vec{k}_2$

$$\vec{x} = c_1 \begin{bmatrix} k_1 \\ \end{bmatrix} e^{\lambda t} + c_2 \begin{bmatrix} k_2 \\ \end{bmatrix} e^{\lambda t}$$

③  $\vec{x}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \vec{x} \quad \lambda, \vec{k} \Rightarrow \text{Find } \vec{p}$

$(A - \lambda I) \vec{p} = \vec{k}$

$$\vec{x}' = c_1 \vec{k} e^{\lambda t} + c_2 \left[ \vec{k} t + \vec{p} \right] e^{\lambda t}$$

Exam  $1\frac{1}{2}$  hr from 8:30 + 10:30

### Series

- Find a series sol to an ODE
- Manipulate a series, including making it look recognizable.

$$\sum_{n=0}^{\infty} x^n$$

$\omega_{nr} \rightarrow$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

### Laplace Transform

- Solving an IVP
- Find Laplace transforms using table, particularly step function
- Find  $\mathcal{L}^{-1}$ , including doing partial fraction, results w/ step function
- Find a Laplace transform from the definition.

Systems - solve  $\vec{x}' = A\vec{x}$

- full eigenvectors
- less than full
- complex

Assn 9 #2

$$\vec{x} = \begin{bmatrix} 3\cos 2t + \sin 2t \\ \cos 2t + 7\sin 2t \end{bmatrix} e^{5t}$$

$$x_1 = (3\cos 2t + \sin 2t) e^{5t}$$

$$x_2 = \dots$$

$$w = f(z)$$

$$y = f(x)$$

