$$y = a_0 + a_1 x + a_2 x^2 + \cdots = \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

Turn into a series whose terms involve x^n.

Let $m = n-2$

$\to n = m+2$

$\to$ when $n=2$, $m=0$

$\to$ when $n=\infty$, $m=\infty$

$$\sum_{m=0}^{\infty} (m+2)\left[(m+2)-1\right] a_{m+2} x^m$$

$$\sum_{m=0}^{\infty} (m+2)(m+1) a_{m+2} x^m$$

OR... $$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n$$

$$y' - 5y = 0 \qquad \text{Assume} \quad y = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y' - 5y = \sum_{n=1}^{\infty} n a_n x^{n-1} - 5 \sum_{n=0}^{\infty} a_n x^n$$

$$\boxed{\sum a_n x^n + \sum b_n x^n = \sum (a_n + b_n) x^n}$$

$m = n-1, \; n = m+1$

$$= \sum_{m=0}^{\infty} (m+1) a_{m+1} x^m - \sum_{m=0}^{\infty} 5 a_m x^m$$

$$= \sum_{m=0}^{\infty} \underbrace{\left[(m+1) a_{m+1} - 5 a_m\right]}_{\text{Each must be zero for this to be true.}} x^m = 0$$

$$(m+1) a_{m+1} - 5 a_m = 0$$

$$\boxed{a_{m+1} = \frac{5 a_m}{m+1} = \frac{5}{m+1} a_m}$$

$$a_0 = a_0$$

$m=0$

$$a_1 = \frac{5}{0+1} a_0 = 5 a_0$$

$m=1$

$$a_2 = \frac{5}{1+1} a_1 = \frac{5}{2} a_1 = \frac{5}{2}(5 a_0) = \frac{5^2}{2} a_0$$

$m=2$

$$a_3 = \frac{5}{2+1} a_2 = \frac{5}{3} a_2 = \frac{5}{3} \cdot \frac{5^2}{2} a_0 = \frac{5^3}{3 \cdot 2} a_0$$

$$y = a_0 + 5 a_0 x + \frac{5^2}{2!} a_0 x^2 + \frac{5^3}{3!} a_0 x^3 + \cdots$$

$$= a_0 \left(1 + 5x + \frac{5^2}{2!} x^2 + \frac{5^3}{3!} x^3 + \cdots \right)$$

$$= a_0 \sum_{n=0}^{\infty} \frac{5^n}{n!} x^n$$