$y' - 5y = 0 \quad$ Assume $y = \sum_{n=0}^{\infty} a_n x^n$

$a_1 = 5a_0$, $a_2 = \frac{5^2}{2} a_0$, $a_3 = \frac{5^3}{3 \cdot 2} a_0$

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

$= a_0 + 5a_0 x + \frac{5^2}{28} a_0 x^2 + \cdots$

$= a_0 \left(1 + 5x + \frac{5^2}{28} x^2 + \cdots \right)$

$+ a_1 \left(\qquad \qquad \right)$

$$y'' - xy' + 2y = 0 \qquad y = \sum_{n=0}^{\infty} a_n x^n$$

$$xy' = x\sum_{n=1}^{\infty} n a_n \cancel{x^{n-1}}\, x^n$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\overset{=0}{y'' - xy' + 2y} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \underbrace{\sum_{n=1}^{\infty} n a_n x^n + \sum_{n=0}^{\infty} 2 a_n x^n}_{\text{ditto}}$$

$m = n-2$

$n = m+2$

$$= \sum_{m=0}^{\infty} (m+2)(m+1) a_{m+2} x^m \qquad \text{ditto} \qquad n=0$$

$m=0$

$$= \underbrace{2a_2 + 2a_0}_{\text{constant term}} + \sum_{n=1} \underbrace{\left[(n+2)(n+1)a_{n+2} - n a_n + 2a_n\right]}_{\text{must} = 0} x^n$$

$a_2 = -a_0 \Longleftarrow$ constant term must $= 0$

$$(n+2)(n+1) a_{n+2} = n a_n - 2 a_n$$

$$\boxed{a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_n}$$

$$a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_n$$

Let's find some a_n's!

$$a_0 = a_0, \quad a_1 = a_1$$

$n=0$ $\quad a_2 = \frac{-2}{2} a_0 = -a_0$

$n=1$ $\quad a_3 = \frac{-1}{3 \cdot 2} a_1 = -\frac{1}{3!} a_1$

$n=2$ $\quad a_4 = 0$

$n=3$ $\quad a_5 = \frac{1}{5 \cdot 4} a_3 = \frac{1}{5 \cdot 4}\left(-\frac{1}{3 \cdot 2} a_1\right) = -\frac{1}{5!} a_1$

$n=4$ $\quad a_6 = (\text{stuff}) a_4 = 0$

$n=5$ $\quad a_7 = \frac{3}{7 \cdot 6} a_5 = -\frac{3}{7!} a_1$

$$a_8 = a_{10} = a_{12} = \cdots = 0$$

$n=7$ $\quad a_9 = \frac{5}{9 \cdot 8} a_7 = -\frac{5 \cdot 3}{9!} a_1$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$= a_0 + a_1 x - a_0 x^2 - \frac{1}{3!} a_1 x^3 - \frac{1}{5!} a_1 x^5$$

$$- \frac{3}{7!} a_1 x^7 - \frac{5 \cdot 3}{9!} a_1 x^9 - \cdots$$

$$= a_0(1 - x^2) + a_1\left(x - \frac{1}{3!} x^3 - \frac{1}{5!} x^5 - \frac{3}{7!} x^7 - \frac{5 \cdot 3}{9!} x^9 - \cdots\right.$$

Apr 29-9:19 AM