$$\boxed{f(x) = x^2 - 5x + 3} \quad -\infty < x < \infty$$

$$g(x) = \sqrt{x} \quad x \geq 0$$

$$y = \frac{1}{2}x + 3$$

$$y = 3 + \frac{1}{2}x$$

$$f(x) = 3 - 5x + x^2$$

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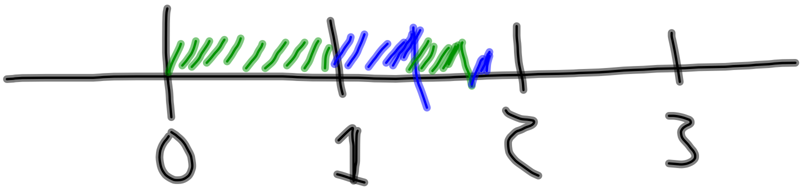
$$y = \underbrace{1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \frac{1}{16}x^4 + \cdots}_{\text{Power series}}$$

$x=0$: $y = 1 + 0 + 0 + \cdots = 1$

$x=1$: $y = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$

$1 \quad 1\frac{1}{2} \quad 1\frac{3}{4} \quad 1\frac{7}{8}$



$x=2$: $y = 1 + 1 + 1 + \cdots$ Doesn't converge for $x=2$

$x=-2$: $y = 1 - 1 + 1 - 1 + 1 - \cdots$ Doesn't converge for $x=-2$

$1 \quad 0 \quad 1$

$x=-1$: $y = \frac{2}{3}$

$y = 1 + \frac{1}{2}x + \frac{1}{4}x^2 + \cdots$ is a valid function if $-2 < x < 2$

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \cdots$$

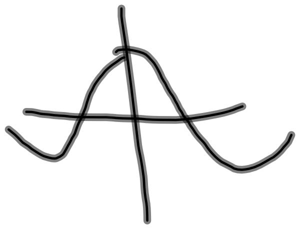
$$0! = 1$$

$$24 = 4 \cdot 3 \cdot 2 \cdot 1 = 4!$$

$$e^x = \frac{1}{0!}x^0 + \frac{1}{1!}x^1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$$

$$\cos x = 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots$$

$$f(-x) = f(x)$$



$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots$$

$$e^x = 1 + x + \frac{1}{2}x^2 + \cdots + \frac{1}{n!}x^n + \cdots$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n \qquad n = 0, 1, 2, 3, \ldots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$$

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$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \cdots = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$$

$$e^{-2x} = 1 + (-2x) + \frac{1}{2}(-2x)^2 + \frac{1}{3!}(-2x)^3 + \cdots$$

$$= 1 - 2x + \frac{2^2}{2}x^2 - \frac{2^3}{3!}x^3 +$$

Expanded form

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} x^n$$

Summation form

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \qquad e^{-2x} = \sum_{n=0}^{\infty} \frac{1}{n!}(-2x)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} x^n$$

General Power series

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots = \sum_{n=0}^{\infty} a_n x^n$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$$

$$y'' = 2a_2 + 3\cdot 2a_3 x + 4\cdot 3 a_4 x^2 + \cdots$$

Solve $\boxed{y' + 5y = 0}$ $\quad y' = -5y$

Guess $y = a_0 + a_1 x + a_2 x^2 + \cdots$ $\quad y = e^{-5x}$

$y' = a_1 + 2a_2 x + 3a_3 x^2 + \cdots$ $\quad y' = -5e^{-5x} = -5y$

$$y' + 5y = (a_1 + 2a_2 x + 3a_3 x^2 + \cdots) + 5(a_0 + a_1 x + a_2 x^2 + \cdots)$$

$$0 = (a_1 + 5a_0) + (2a_2 + 5a_1)x + (3a_3 + 5a_2)x^2 + \cdots$$

$a_1 + 5a_0 = 0$ $\quad 2a_2 + 5a_1 = 0$ $\quad 3a_3 + 5a_2 = 0.$

$a_1 = -5a_0$ $\quad 2a_2 = -5a_1$

$2a_2 = (-5)(-5)a_0$

$$a_2 = \frac{(-5)^2}{2} a_0$$

$3a_3 = -5a_2$

$$3a_3 = -5\left(\frac{(-5)^2}{2} a_0\right)$$

$$a_3 = \frac{(-5)^3}{3\cdot 2} a_0$$

guess

$$a_4 = \frac{(-5)^4}{4!} a_0$$

$$a_n = \frac{(-5)^n}{n!} a_0$$

Guess $y = a_0 + a_1 x + a_2 x^2 + \cdots$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n \qquad = \sum_{n=0}^{\infty} \frac{(-5)^n}{n!} a_0 x^n = a_0 \sum_{n=0}^{\infty} \frac{(-5)^n}{n!} x^n$$

$$= a_0 \sum_{n=0}^{\infty} \frac{1}{n!} (-5x)^n$$

$$= a_0 e^{-5x}$$

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