$$x_1' = -7x_1 + 8x_2$$
$$x_2' = -4x_1 + 5x_2$$

Where x_1 and x_2 are functions of time.

1. Solve the first equation for x_2:
2. Find x_2' using your answer to 1.
3. Substitute your answers to 1 and 2 into the second equation. Combine like terms and solve the resulting 2nd order ODE.

Apr 8-8:53 AM

$$x_1' = -7x_1 + 8x_2$$
$$x_2' = -4x_1 + 5x_2$$

$$x_2 = \frac{x_1' + 7x_1}{8}$$

$$x_2' = \frac{1}{8}(x_1'' + 7x_1')$$

$$\frac{1}{8}x_1'' + \frac{7}{8}x_1' = -4x_1 + 5x_2$$

$$\frac{1}{8}x_1'' + \frac{7}{8}x_1' = -4x_1 + \frac{5}{8}x_1' + \frac{35}{8}x_1$$

multiply by 8

$$x_1'' + 7x_1' = -32x_1 + 5x_1' + 35x_1$$

$$x_1'' + 2x_1' - 3x_1 = 0$$

$y = e^{rt}$

$$r^2 + 2r - 3 = 0$$
$$(r+3)(r-1) = 0$$

$r = -3,\ r = 1$

$$x_1 = c_1 e^{-3t} + c_2 e^{t}$$

Apr 8-9:03 AM

$A\vec{x} = \lambda\vec{x}$ eigenvalue/eigenvector equation.

everybody else $\downarrow$

$A\vec{x} - \lambda\vec{x} = \vec{0}$

$(A - \lambda I)\vec{x} = \vec{0}$

$B\vec{x} = \vec{0}$

Khan $\searrow$

$\lambda\vec{x} - A\vec{x} = \vec{0}$

$(\lambda I - A)\vec{x} = \vec{0}$

$\rightarrow \vec{x} \neq \vec{0}$

$\rightarrow$ only has other sols if $\det B = 0$

$A = \begin{bmatrix} -7 & 8 \\ -4 & 5 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} -7 & 8 \\ -4 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} -7-\lambda & 8 \\ -4 & 5-\lambda \end{bmatrix}$$

Khan: $\begin{bmatrix} \lambda+7 & -8 \\ 4 & \lambda-5 \end{bmatrix}$

$$0 = \det \begin{bmatrix} -7-\lambda & 8 \\ -4 & 5-\lambda \end{bmatrix} = (-7-\lambda)(5-\lambda) - (-4)(8)$$

$$= -35 + 2\lambda + \lambda^2 + 32$$

$$= \lambda^2 + 2\lambda - 3$$

$$= (\lambda+3)(\lambda-1)$$

$\boxed{\lambda = -3, 1}$

eigenvalues

$(A-\lambda I)\vec{x} = \vec{0}$ $\lambda = -3, 1$, find corresponding $\vec{x}$ vectors

$$A-\lambda I = \begin{bmatrix} -7-\lambda & 8 \\ -4 & 5-\lambda \end{bmatrix}$$

$\lambda = -3$: $\begin{bmatrix} -4 & 8 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -4x_1 + 8x_2 = 0$

$x_1 - 2x_2 = 0$

$x_1 = 2x_2$

$\vec{x} = \begin{bmatrix} 2x_2 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is <u>an</u> eigenvector associated with $\lambda = -3$

$$x_1' = -7x_1 + 8x_2$$
$$x_2' = -4x_1 + 5x_2$$

Let $\vec{x} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$

$\vec{x}(t)$

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\boxed{\vec{x}' = A\vec{x}}$$ Matrix form of system

$\underline{\underline{x' = kx}}$ $x = x(t)$

$\lambda = k$

Guess $x = e^{\lambda t}$

$\underline{\underline{x' = \lambda e^{\lambda t} = \lambda x}}$

$$\boxed{x = ce^{\lambda t}}$$
$$x' = \lambda c e^{\lambda t} = \lambda x$$

Guess $\vec{x} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} e^{\lambda t} = \vec{c} e^{\lambda t}$

Apr 8-9:14 AM

$\vec{x}' = A\vec{x}$ Guess $\vec{x} = \vec{c}e^{\lambda t}$

$\vec{x}' = \lambda\vec{c}e^{\lambda t}$

$\lambda\vec{c}\cancel{e^{\lambda t}} = A\vec{c}\cancel{e^{\lambda t}}$

$\lambda\vec{c} = A\vec{c}$ $\vec{c}$ is an eigenvector, with eigenvalue λ.

$\lambda_1 = 1$, $\vec{c}_1 = d_1\begin{bmatrix}1\\1\end{bmatrix}$

$\vec{c}$ is eigenvector

$\lambda_2 = -3$, $c_2 = d_2\begin{bmatrix}2\\1\end{bmatrix}$ $A(k\vec{c}) = kA\vec{c} = k\lambda\vec{c} = \lambda(k\vec{c})$

$$\boxed{\begin{array}{l}\text{Solution:}\\ \vec{x} = d_1\begin{bmatrix}1\\1\end{bmatrix}e^{t} + d_2\begin{bmatrix}2\\1\end{bmatrix}e^{-3t}\end{array}}$$

$x_1 = d_1e^{t} + 2d_2e^{-3t}$

$x_2 = d_1e^{t} + d_2e^{-3t}$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = \begin{bmatrix}d_1e^{t} + 2d_2e^{-3t}\\ d_1e^{t} + d_2e^{-3t}\end{bmatrix}$$