From a system $\vec{x}' = A\vec{x}$ and the eigenvalues + eigenvectors of A,

- sketch a portion of the direction field
- sketch the phase portrait
- classify the critical point at the origin
- classify the stability

Remember, $\vec{x}' = A\vec{x}$ means

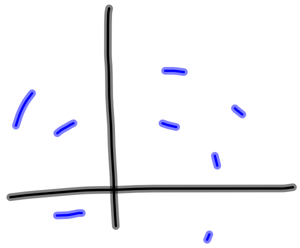
is $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

x_1, x_2 are functions of time.

$$x_1' = ax_1 + bx_2$$
$$x_2' = cx_1 + dx_2$$

$$\vec{x}' = \begin{bmatrix} -3 & -\frac{1}{2} \\ -2 & -3 \end{bmatrix} \vec{x}$$

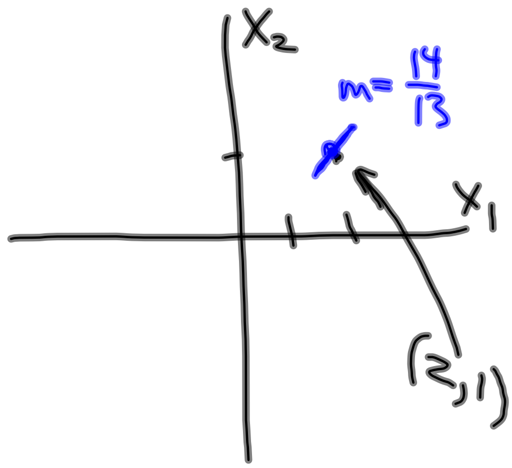
Direction field?



At $(2,1) \sim \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} -3 & -\frac{1}{2} \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{13}{2} \\ -7 \end{bmatrix} = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix}$$

$$\frac{dx_2/dt}{dx_1/dt} = \frac{dx_2}{dx_1} = \frac{-7}{-\frac{13}{2}} = \frac{14}{13}$$



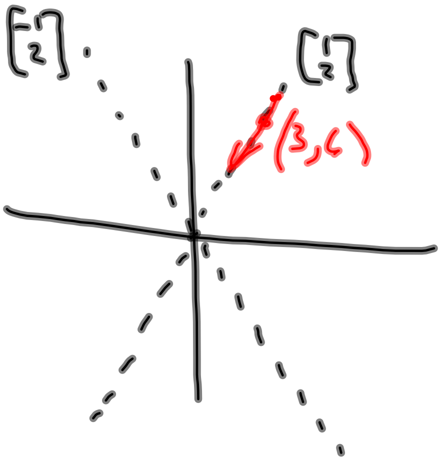
Phase portrait for $\vec{x}' = \left[\ A\ \right] \vec{x}$

$\lambda = -2, -1 \quad \vec{k} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = -2\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$

$A\begin{bmatrix} -5 \\ -10 \end{bmatrix} = -2\begin{bmatrix} -5 \\ -10 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

<u>Step 1</u> put in eigenspaces as dashed lines:

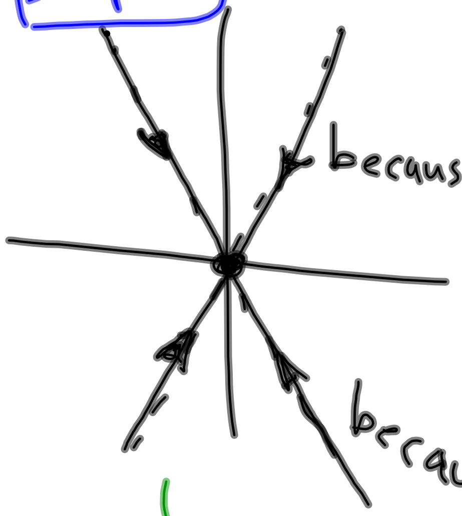


Suppose $\vec{x}(0) = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

$\vec{x}'(0) = A\begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} -6 \\ -12 \end{bmatrix} = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix}$

$\vec{x}' = A\vec{x}$, $\lambda = -2, -1$ $\vec{k} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$\boxed{\text{Step 2}}$

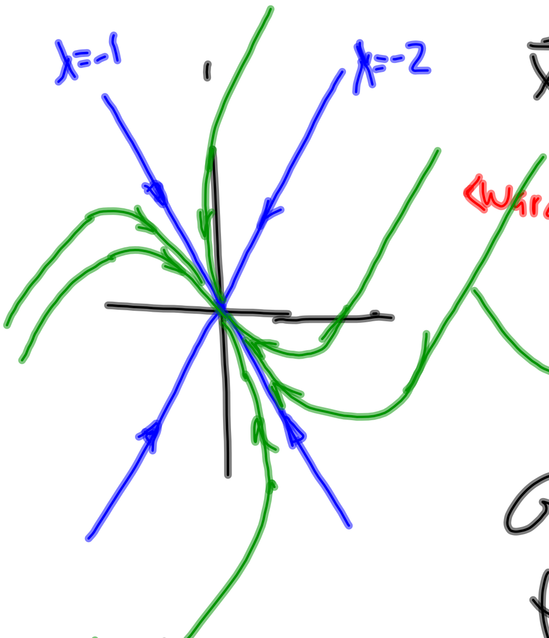


$\vec{x} = \underbrace{c_1 \vec{k}_1 e^{\lambda_1 t}} + c_2 \vec{k}_2 e^{\lambda_2 t}$

this scalar is getting

4 trajectories Others?

$\vec{x}\,' = A\vec{x}$, $\lambda = -2, -1$ $\quad \vec{k} = \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}-1\\2\end{bmatrix}$



$$\vec{x} = c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-2t} + c_2\begin{bmatrix}-1\\2\end{bmatrix}e^{-t} \quad (1)$$

ward in time

Other trajectoies?

Pick a point.

Going "backward" in time, $t<0$ and (1) becomes

$$\vec{x} = \underbrace{c_1\vec{k}_1 e^{2(pos)}}_{\text{dominant term}} + c_2\vec{k}_2 e^{1(pos)}$$

Going forward in time,

$$\vec{x} = c_1 k_1 e^{2(neg)} + \underbrace{c_2 k_2 e^{1(neg)}}_{\text{dominant term}}$$