$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ has $\lambda = 1 \pm i$, $\vec{k} = \begin{bmatrix} 1 \\ i \end{bmatrix}, \begin{bmatrix} 1 \\ -i \end{bmatrix}$

① Sketch the phase portrait for $\vec{x}' = A\vec{x}$.

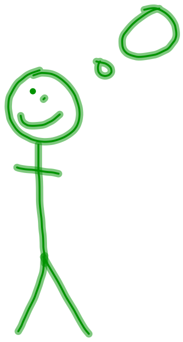
② Choose one for the origin: nodal sink, nodal source, spiral sink, spiral source, center.

③ Choose one for stability: unstable, asymptotically stable, neutrally stable.

$A=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ has $\lambda = 1 \pm i$, $\vec{k}=\begin{bmatrix}1\\ i\end{bmatrix},\begin{bmatrix}1\\ -i\end{bmatrix}$

$\vec{x}' = [A]\vec{x}$

$e^{(1+i)t} = e^{t}(\cos t + i\sin t)$



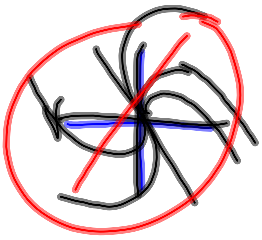
spiralling outward

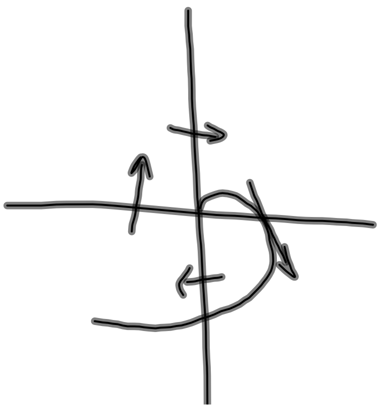
There are no eigenvector trajectories

Spirals or ellipses

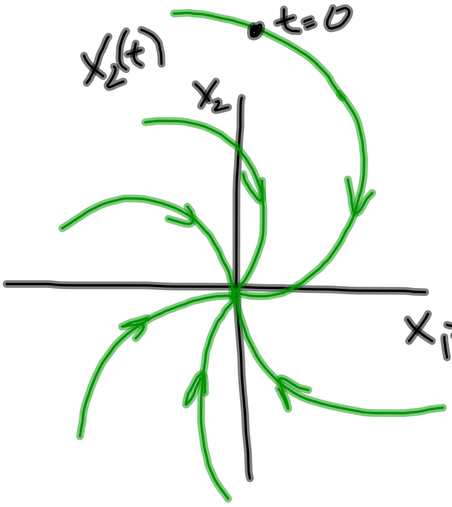
Choose $\vec{x} = \begin{bmatrix}1\\ 0\end{bmatrix}$ to determine direction of spiral.

$\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix} = \begin{bmatrix}1\\ -1\end{bmatrix}$

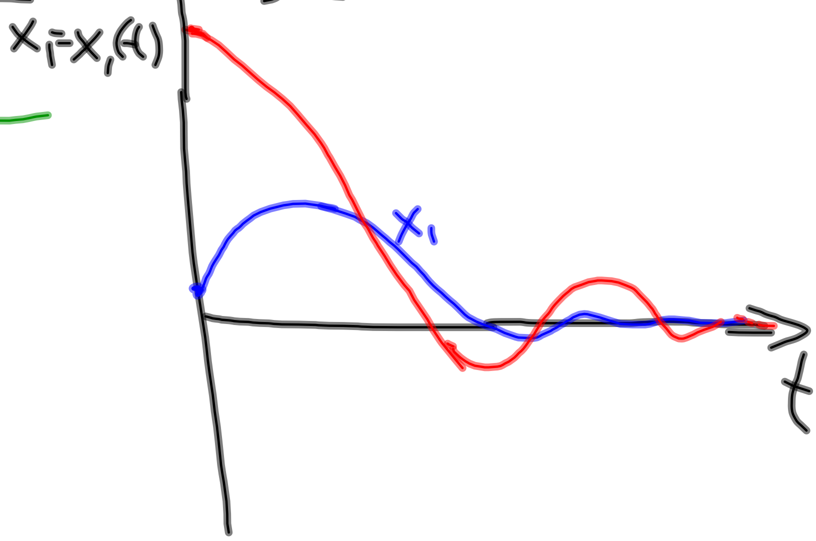


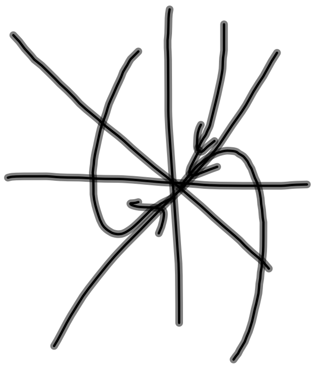


Spiral source
unstalbe



asymptotically stable





$\vec{x}' = A\vec{x} + ?$

$y'' + y' - 6y = 5t - 3 \qquad y = y_h + y_p$

Homogeneous:

$r^2 + r - 6 = 0$

$(r+3)(r-2) = 0$

$r = -3, 2$

$$\boxed{y_h = c_1 e^{-3t} + c_2 e^{2t}}$$

Guess

$y_p = At + B$

$y_p' = A$

$y_p'' = 0$

undetermined coefficients

$0 + A - 6(At + B) = 5t - 3$

$-6At + (A - 6B) = 5t - 3$

$-6A = 5 \qquad A - 6B = -3$

$A = -\frac{5}{6} \qquad -\frac{5}{6} - 6B = -3$

$-5 - 36B = -18$

$-36B = -13$

$B = \frac{13}{36}$

$$\boxed{y_p = -\frac{5}{6}t + \frac{13}{36}}$$

$$\boxed{y = y_h + y_p = c_1 e^{-3t} + c_2 e^{2t} - \frac{5}{6}t + \frac{13}{36}}$$

$\vec{x}' = A\vec{x} + \vec{f}$

OR

$\vec{x}'(t) = A\vec{x}(t) + \vec{f}(t)$

$y' - 3y = \sin 5t$

$y' = 3y + \underbrace{\sin 5t}_{f(t)}$

Non-homogeneous system.

Solve $\vec{x}' = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\vec{x} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} e^{-2t}$ (1)

1 Solve $\vec{x}' = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\vec{x}$ to get $\vec{x}_h$

$$\boxed{\vec{x}_h = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t}} \Longrightarrow$$

See next page for eigenvalues/eigenvectors

2 What is $\vec{x}_p$? Guess $\vec{x}_p = \begin{bmatrix} a \\ b \end{bmatrix} e^{-2t}$ and put into (1)

$\vec{x}_p' = -2\begin{bmatrix} a \\ b \end{bmatrix} e^{-2t}$ $\quad \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\vec{x}_p = \begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} e^{-2t} = \begin{bmatrix} 2a+3b \\ 2a+b \end{bmatrix} e^{-2t}$

$$-2\begin{bmatrix} a \\ b \end{bmatrix} \cancel{e^{-2t}} = \begin{bmatrix} 2a+3b \\ 2a+b \end{bmatrix} \cancel{e^{-2t}} + \begin{bmatrix} -2 \\ 1 \end{bmatrix} \cancel{e^{-2t}}$$

$$\begin{bmatrix} -2a \\ -2b \end{bmatrix} = \begin{bmatrix} 2a+3b-2 \\ 2a+b+1 \end{bmatrix}$$

$-2a = 2a+3b-2$

$-2b = 2a+b+1$ $\Longrightarrow$ solve for a and b

$4a + 3b = 2$

$2a + 3b = -1$ } subtract

$2a = 3$

$a = \frac{3}{2}$

$2\left(\frac{3}{2}\right) + 3b = -1$

$3b = -4$

$b = -\frac{4}{3}$

$$\boxed{\vec{x}_p = \begin{bmatrix} \frac{3}{2} \\ -\frac{4}{3} \end{bmatrix} e^{-2t}}$$

3 General solution:

$$\vec{x} = \vec{x}_h + \vec{x}_p = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 3 \\ 2 \end{bmatrix} e^{4t} + \begin{bmatrix} \frac{3}{2} \\ -\frac{4}{3} \end{bmatrix} e^{-2t}$$

May 16-9:33 AM

$$\begin{bmatrix} 2 & 3 \\ 2 & 1 \end{bmatrix}$$

$$|(A-\lambda I)| = 0$$

$$\begin{vmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{vmatrix} = (2-\lambda)(1-\lambda)-6$$

$$= 2-3\lambda+\lambda^2-6$$

$$\lambda^2-3\lambda-4=0 \quad (\lambda-4)(\lambda+1$$

$$\lambda = -1, 4$$

let $\lambda = -1$,

$$\begin{bmatrix} 3 & 3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \vec{0}$$

$$\vec{k}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$-2k_1+3k_2=0$

$2k_1-3k_2=0$

$2k_1=3k_2$

$k_1=\frac{3}{2}k_2$

Let $k_2=2$

$k_1=3$

$$\lambda = 4 \quad \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = 0$$

$$\vec{k} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$-2k_1+3k_2=0$$

$$\vec{k}_2 = \begin{bmatrix} 1 \\ 2/3 \end{bmatrix} \text{ or } \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Assignment 19

Due 5/17
3PM

Solve

$$\vec{x}' = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 0 & 5 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} e^{4t}$$

Use Wolfram Alpha to get eigenvalues & eigenvectors.