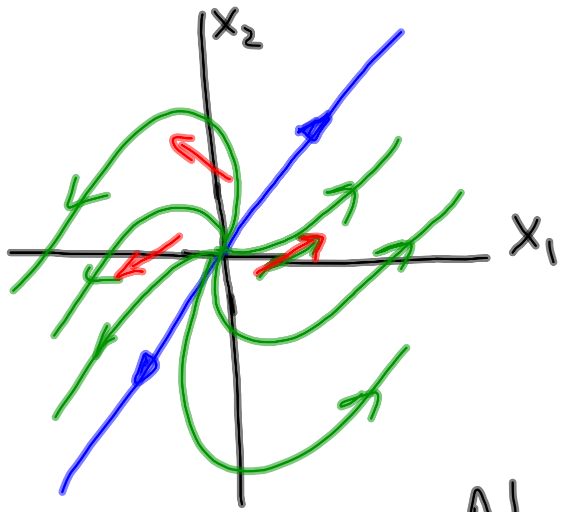
① $A = \begin{bmatrix} 3 & -1 \\ 1 & 1 \end{bmatrix}$ has eigenvalue 2 with multiplicity 2 and only one eigenvector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Sketch the phase portrait for $\vec{x}' = A\vec{x}$.



$$\underset{\vec{x}}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} \Rightarrow \underset{\vec{x}'}{\begin{bmatrix} 3 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Nodal source, unstable

② Repeat ① for $\vec{x}' = B\vec{x}$. $B = \begin{bmatrix} 2 & 8 \\ -1 & -2 \end{bmatrix}$

has eigenvalues $\underbrace{\pm 2i}$ and eigenvectors

$\vec{k} = \begin{bmatrix} 2 \pm 2i \\ -1 \end{bmatrix}$.

real part is zero

Solutions contain

sin2t + cos2t

$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \Longrightarrow \begin{bmatrix} 8 \\ -2 \end{bmatrix}$

The origin is a center.

Neutrally stable

$$y'' + 4y' + 8y = \sin 5t$$

$$r^2 + 4r + 8 = 0 \qquad y_p = A\sin 5t + B\cos 5t$$

$$r = -2 \pm 2i$$

$$y_h = e^{-2t}(c_1 \sin 2t + c_2 \cos 2t)$$

$$= e^{-2t}\left(-\frac{1293}{1378}\sin 2t + \frac{709}{689}\cos 2t\right)$$

$$\vec{x}' = A\vec{x} + \vec{f}$$

- $\vec{y}_p =$

- Solutions to $\vec{x}' = A\vec{x}$ $\quad \vec{k}_1 e^{\lambda_1 t}, \vec{k}_2 e^{\lambda_2 t}, \dots$

$$\vec{X}_h = c_1\vec{X}_1 + c_2\vec{X}_2 + \cdots \qquad \vec{X}_1, \vec{X}_2$$

$$\Phi(t) = \begin{bmatrix} \vdots & \vdots & \\ \vec{X}_1 & \vec{X}_2 & \cdots \\ \vdots & \vdots & \end{bmatrix}$$ Fundamental matrix

$$\vec{X}_p = \Phi \underbrace{\int \underbrace{\Phi^{-1}\vec{f}}_{\text{vector}}\, dt}_{\text{vector}} \qquad \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-3t} = \begin{bmatrix} e^{-3t} \\ 2e^{-3t} \end{bmatrix}$$

$$X = \begin{bmatrix} e^{-2t} & e^{-t} \\ 2e^{-2t} & 3e^{-t} \end{bmatrix}$$

$$\frac{1}{e^{\infty}} = e^{-\infty}$$

$$X^{-1} = \begin{bmatrix} 3e^{2t} & -e^{2t} \\ -2e^{t} & e^{t} \end{bmatrix}$$

$$X^{-1}\vec{f} =$$

$$\int X^{-1}\vec{f}\,dt =$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} a+2b \\ c+2d \end{bmatrix} = 1\begin{bmatrix} a \\ c \end{bmatrix} + 2\begin{bmatrix} b \\ d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$$

May 17-9:06 AM