$$x^{\lambda}\left[\underbrace{(\lambda^2-1)}_{=0}a_0+\underbrace{(\lambda^2+2\lambda)a_1x}_{=0}+\underbrace{\sum_{n=2}^{\infty}\bigcirc\, x^n}\right]=0$$

$(\lambda^2-1)a_0=0$

$\boxed{\lambda=\pm 1}$

$$\boxed{\begin{aligned}&\lambda=1:\ 3a_1x=0 \Rightarrow a_1=0\\ &\lambda=-1:\ -a_1x=0 \Rightarrow a_1=0\end{aligned}}$$

$$n(n+1)a_n+(n+1)a_n+a_{n-2}-a_n=0$$

$$[n(n+1)+(n+1)-1]a_n+a_{n-2}=0$$

$(n+1)(n+1)-1$

$(n+1)^2-1$

$n^2+n+n\cancel{+1}\cancel{-1}$

n^2+2n

$n(n+2)$

$$a_n=\frac{-1}{[n(n+2)]}a_{n-2}$$

$a_0=1$

$a_2=-\frac{1}{2\cdot 4}$

$a_4=-\frac{1}{4\cdot 6}a_2=\frac{1}{2\cdot 4\cdot 4\cdot 6}$

$a_6=-\frac{1}{6\cdot 8}$ $=\frac{-1}{2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 8}$

$\frac{1}{2\cdot 4\cdot 4\cdot 6\cdot 6\cdot 8\cdot 8\cdot 10}$

Assumption

$y=x^{\lambda}\sum$

May 22-9:06 AM

Exam 2

* 8:30-10:30 $1\frac{1}{2}$ hour in that period
* You can use the formula sheet
* Concept Map for Series Sols
* List of topics online
* Group A: Phase portraits

  Group B: Series Sols

  ⋮

  Matrix Sols

You **must** be able to solve

$\vec{x}' = A\vec{x} + \vec{f}$, $\vec{x}(0) = \vec{c}$ start to finish.

May 22-9:20 AM

Series Solutions

$a(x)y'' + b(x)y' + c(x)y = 0$

a, b, c non-constant

If $x=0$ is an ordinary, solution is of form $y = \sum_{n=0}^{\infty} a_n x^n$. Assn. 11

If $x=0$ is a regular singular point, then solution is $y = x^{\lambda} \sum_{n=0}^{\infty} a_n x^n$ (method of Frobenius)

Either way, the solution is

$y = c_1 y_1(x) + c_2 y_2(x)$

$$x^2(1-x^2)y'' + \frac{2}{x}y' + 4y = 0$$

$x^2(1+x)(1-x)$

$x = u+1$

$u = x-1$

Singular points: $x = 0, 1, -1$

not regular (0), regular (1), regular (−1)

$$y'' + \frac{2}{x^3(1+x)(1-x)}y' + \frac{4}{x^2(1+x)(1-x)}y = 0$$

Whole new problem

$$x^2(1-x^2)y'' + \frac{2x}{(1-x)}y' + 4y = 0$$

Singular points

$0, 1, -1$

regular, not regular, regular

$$y'' + \frac{2\cancel{x}}{x^{\cancel{2}}(1+x)\underbrace{(1-x)(1-x)}_{}}y' + \frac{4}{x^2(1+x)(1-x)}y = 0$$

1 is not regular