Let $A = \begin{bmatrix} -4 & 6 \\ -3 & 5 \end{bmatrix}$, $\vec{x}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\vec{x}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

a) Find $A\vec{x}_1$ and $A\vec{x}_2$. What can you conclude?

$$A\vec{x}_1 = \begin{bmatrix} -2 \\ -1 \end{bmatrix} \qquad A\vec{x}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$\vec{x}_1$ is an eigenvector because $A\vec{x}_1$ is a scalar multiple of $\vec{x}_1$.
So $\lambda_1 = -1$

$x_2$ has eigenvalue $\lambda_2 = 2$.

b) Find $AP$, where $P = \begin{array}{cc} \vec{x}_1 & \vec{x}_2 \\ \multicolumn{2}{c}{\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}} \end{array}$

$$AP = \begin{bmatrix} -2 & 2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} A\vec{x}_1 & A\vec{x}_2 \end{bmatrix} = \begin{bmatrix} | & | \\ \lambda_1\vec{x}_1 & \lambda_2\vec{x}_2 \\ | & | \end{bmatrix}$$

c) Use your calculator to find $A^5$.

$$A^5 = \begin{bmatrix} -34 & 66 \\ -33 & 65 \end{bmatrix}$$

$A$, $P$ has columns that are the eigenvectors, then

$$\underset{[\vec{x}_1\ \vec{x}_2]}{AP} = \begin{bmatrix} A\vec{x}_1 & A\vec{x}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1\vec{x}_1 & \lambda_2\vec{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \vec{x}_1 & \vec{x}_2 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}}_{D}$$

$$\begin{bmatrix} 1 & 2 \\ 3. & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 13 & 3 \\ 29 & 5 \end{bmatrix}$$

$$3\begin{bmatrix} 1 \\ 3 \end{bmatrix} + 5\begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix} + \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 13 \\ 29 \end{bmatrix}$$

$$AP = PD \longrightarrow P^{-1}AP = D$$

$$\longrightarrow \boxed{A = PDP^{-1}}$$ Diagonalizing $A$

$$A = PDP^{-1}$$

$$A^5 = (PDP^{-1})^5$$

$$= PD\cancel{P^{-1}}\cancel{P}D\cancel{P^{-1}}\cancel{P}D\cancel{P^{-1}}\cancel{P}D\cancel{P^{-1}}\cancel{P}DP^{-1}$$

$$= PDDDDDP^{-1}$$

$$= PD^5P^{-1}$$

$$D = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$$

$$D^2 = \begin{bmatrix} 16 & 0 \\ 0 & 81 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix} = \begin{bmatrix} 16+0 & 0+0 \\ 0+0 & 0+81 \end{bmatrix} = \begin{bmatrix} 16 & 0 \\ 0 & 81 \end{bmatrix}$$

$$A = \begin{bmatrix} -4 & 6 \\ -3 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$P = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \qquad D^5 = \begin{bmatrix} -1 & 0 \\ 0 & 32 \end{bmatrix}$$

$$A^5 = PD^5P^{-1}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & 32 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ -32 & 64 \end{bmatrix}$$

$$\boxed{\frac{1}{1} \begin{bmatrix} -34 & 66 \\ -33 & 65 \end{bmatrix} = A^5}$$

$$\begin{array}{ccccccccccc} & a_0 & a_1 & a_2 & a_3 & a_4 & & & & \\ S = & 1, & 1, & 2, & 3, & 5, & 8, & 13, & 21, & \ldots \end{array}$$

$$\underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\vec{x}^0} = \underbrace{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}_{\vec{x}_1} \qquad \vec{x}^1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \begin{array}{l} a_1 \\ \\ \end{array}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \underbrace{\begin{bmatrix} 3 \\ 5 \end{bmatrix}}_{\vec{x}_3} \begin{array}{l} a_3 \\ \\ \end{array} \qquad \vec{x}^2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \begin{array}{l} a_2 \\ \\ \end{array}$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$a_n$ is the 1st coordinate of $\vec{x}^n$

$$\boxed{\vec{x}_n = A^n \vec{x}_0}$$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\det(A - \lambda I) = \det \begin{bmatrix} -\lambda & 1 \\ 1 & 1-\lambda \end{bmatrix} = 0$$

$$= -\lambda(1-\lambda) - 1 = \lambda^2 - \lambda - 1 = 0$$

$$\frac{1 \pm \sqrt{1-4(1)(-1)}}{2} = \frac{1}{2} \pm \frac{\sqrt{5}}{2}$$