Solve each ODE, taking the independent variable to be t:

$y' + 3y = 0$

$y = Ce^{-3t}$ $y' = -3y$

$y' = -3Ce^{-3t}$

$$\underline{y'' + 3y = 0}$$

$r^2 + 3 = 0$

$r^2 = -3$

$r = \pm\sqrt{3}\,i$

$y = c_1 \sin\sqrt{3}\,t + c_2 \cos\sqrt{3}\,t$

Fibonacci Sequence

$a_0, a_1, a_2, a_3, a_4, \dots$ $\quad a_n = a_{n-1} + a_{n-2}$

$0, 1, 1, 2, 3, 5, 8, \dots$ Explicit Formula?

$\vec{x}_n = \begin{bmatrix} a_n \\ a_{n+1} \end{bmatrix}$ $\quad A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ $\quad A\vec{x}_n = \vec{x}_{n+1}$

$\vec{x}_n = A^n \vec{x}_0$ $\quad A = PDP^{-1}$ P has columns eigenvectors of A

D is diagonal with eigenvalues on the diagonal

$A^n = (PDP^{-1})^n = PD^nP^{-1}$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \quad \lambda = \frac{1 \pm \sqrt{5}}{2}$$

$$(A - \lambda I)\vec{c} = \begin{bmatrix} -\frac{1}{2} - \frac{\sqrt{5}}{2} & 1 \\ 1 & \frac{1}{2} - \frac{\sqrt{5}}{2} \end{bmatrix} \vec{c} = \vec{0} \quad \vec{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\left(-\frac{1}{2} - \frac{\sqrt{5}}{2}\right)c_1 + c_2 = 0$$

$$c_2 = \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)c_1 \qquad \vec{z}_1 = \begin{bmatrix} 1 \\ \frac{1}{2} + \frac{\sqrt{5}}{2} \end{bmatrix}$$

let $c_1 = 1 \Longrightarrow$

$$P = \begin{bmatrix} 1 & 1 \\ \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) & \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) \end{bmatrix} \quad D = \begin{bmatrix} \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) & 0 \\ 0 & \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) \end{bmatrix}$$

$$P^{-1} = \frac{1}{\sqrt{5}} \begin{bmatrix} \left(-\frac{1}{2} + \frac{\sqrt{5}}{2}\right) & 1 \\ \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) & -1 \end{bmatrix} \quad \vec{X}_n = PD^nP^{-1}\vec{X}_0$$

$$\begin{bmatrix} 1 & 1 \\ \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) & \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) \end{bmatrix} \begin{bmatrix} \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^n & 0 \\ 0 & \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)^n \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \vec{X}_n$$

$$\frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 1 \\ \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right) & \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right) \end{bmatrix} \begin{bmatrix} \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^n \\ -\left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)^n \end{bmatrix} = \vec{X}_n = \begin{bmatrix} X_n \\ X_{n+1} \end{bmatrix}$$

$$\frac{1}{\sqrt{5}} \begin{bmatrix} \left(\frac{1}{2} + \frac{\sqrt{5}}{2}\right)^n - \left(\frac{1}{2} - \frac{\sqrt{5}}{2}\right)^n \\ \text{STUFF} \end{bmatrix}$$

$$\Rightarrow a_n = \frac{1}{\sqrt{5}} \left(\frac{(1+\sqrt{5})^n}{2^n} - \frac{(1-\sqrt{5})^n}{2^n} \right)$$

$$a_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

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$$x_1' = x_1 + 2x_2 \qquad x_1(0) = 10$$

$$x_2' = 3x_1 + 2x_2 \qquad x_2(0) = 5$$

Idea: $\vec{x}' = A\vec{x} \qquad \vec{x}(0) = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$

$$\vec{x}' = PDP^{-1}\vec{x}$$

$$P^{-1}\vec{x}' = \cancel{P^{-1}P}DP^{-1}\vec{x}$$

$$(P^{-1}\vec{x})' = D(P^{-1}\vec{x}) \Longrightarrow \text{Let } \underline{\vec{y} = P^{-1}\vec{x}}$$

$$\vec{y}(0) = P^{-1}\vec{x}(0)$$

$$\vec{y}' = D\vec{y}, \quad \vec{y}(0) = [\ ]$$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \lambda_1 y_1 \\ \lambda_2 y_2 \end{bmatrix}$$

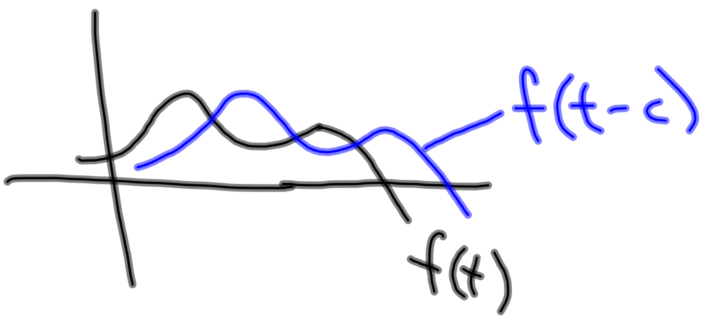
$y_1' = \lambda_1 y_1 \qquad y_1 = c_1 e^{\lambda_1 t}$

$y_2' = \lambda_2 y_2 \qquad y_2 = c_2 e^{\lambda_2 t}$

$$\vec{x} = P\vec{y}$$

$$\delta_h(t) = \begin{cases} \frac{1}{2h} & \text{for } -h < t < h \\ 0 & \text{elsewhere} \end{cases}$$

$$\int_{-\infty}^{\infty} \delta_h(t)\,dt = \int_{-h}^{h} \frac{1}{2h}\,dt$$



$$\int_0^{\infty} \delta_1(t-3)\,dt$$