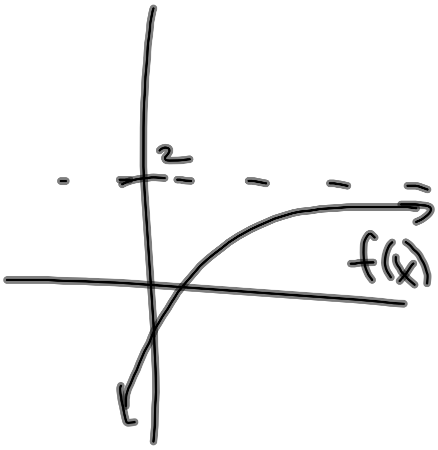
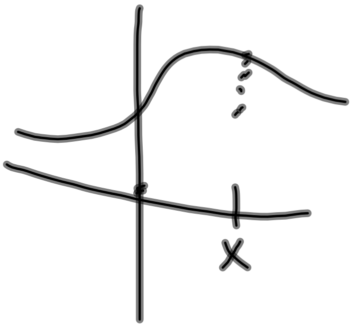


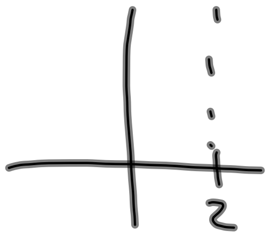
f(x) converges to 2

$$\lim_{x\to\infty} f(x) = 2$$



$$|f(x) - 2| \to 0 \text{ as } x \to \infty$$

$$f(x) = \frac{1}{x-2}$$



$$e^x = 1 + (x-0) + \frac{1}{2}(x-0)^2 + \frac{1}{3}x^3 + \cdots$$

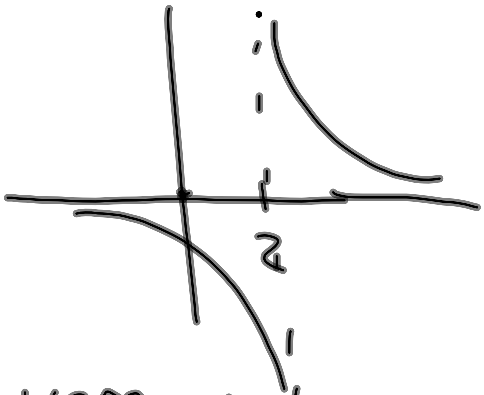
$$e^2 = \sum_{j=0}^{n} \frac{1}{n!} 2^n$$

$$\left| \sum_{j=0}^{n} \frac{1}{n!} 2^n - e^2 \right| \to 0 \text{ as } n \to \infty$$

$f(x)$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

$$f(x) = \frac{1}{x-2}$$



$R = 2$

Interval of convergence

$(0-R, 0+R)$

$$a(x)y'' + b(x)y' + c(x)y = 0$$

Seek 2 independent solutions $y_1(x), y_2(x)$. The most general solution is $y(x) = c_1 y_1(x) + c_2 y_2(x)$.

$x'' + 5x' + 6x = 0$ $\quad x = x(t)$

$r^2 + 5r + 6 = 0 \Rightarrow r = -2, -3$

Assume $y = e^{rt}$ $\quad x_1(t) = e^{-2t}, x_2(t) = e^{-3t}$

No

Assume $\boxed{e^{-2t} = c_1 e^{-3t}}$ for all t $\quad x(t) = c_1 e^{-2t} + c_2 e^{-3t}$

$e^{3t} e^{-2t} = c_1$ $\quad \cancel{c_1 = e^{t}}$

$x'' + 6x' + 9x = 0$

$r^2 + 6r + 9 = 0 \Rightarrow r = -3$ $\quad x_1(t) = e^{-3t}$

$x_2(t) = ?$

Assn 11 2(b)

$a_0 = \cancel{a_0}\ 1$

$a_1 = \cancel{a_1}\ 1$

$a_2 = 2\cancel{a_0}$

$a_3 = \frac{5}{3}\cancel{a_1}$

$a_4 = 3\cancel{a_0}$

$a_5 = \frac{2}{3}\cancel{a_1}$

$y = a_0 + a_1 x + 2a_0 x^2 + \frac{5}{3}a_1 x^3 + 3a_0 x^4 + \frac{7}{3}a_1 x^5 + \cdots$

$$y = a_0\underbrace{\left(1 + 2x^2 + 3x^4 + 4x^6 + \cdots\right)}_{y_1(x)} + a_1\underbrace{\left(x + \frac{5}{3}x^3 + \frac{7}{3}x^5 + \cdots\right)}_{y_2(x)}$$

$a_{n+2} = \frac{n+4}{n+2}a_n$ (?)

May 3-9:21 AM

Method of Frobenius

$3xy'' + y' - y = 0$ $\quad$ $x=0$ is singular, but regular

We want a series solution about $x=0$.

Guess $y = x^\lambda \sum_{n=0}^{\infty} a_n x^n$

$y = \sum_{n=0}^{\infty} a_n x^n$

$y' = \sum_{n=0}^{\infty} a_n n x^{n-1}$

$y = \sum_{n=0}^{\infty} a_n x^{\lambda+n}$

$y' = \sum_{n=0}^{\infty} a_n (\lambda+n) x^{\lambda+n-1} = x^\lambda \sum_{n=0}^{\infty} a_n (\lambda+n) x^{n-1}$

$y'' = \sum_{n=0}^{\infty} a_n (\lambda+n)(\lambda+n-1) x^{\lambda+n-2}$

$= x^\lambda \sum_{n=0}^{\infty} a_n (\lambda+n)(\lambda+n-1) x^{n-2}$

$$3xy'' + y' - y = 0$$

$$3x x^{\lambda} \sum_{n=0}^{\infty} a_n (\lambda+n)(\lambda+n-1) x^{n-2}$$
$$+ x^{\lambda} \sum_{n=0}^{\infty} a_n (\lambda+n) x^{n-1} - x^{\lambda} \sum_{n=0}^{\infty} a_n x^n$$

$$x^{\lambda} \left\{ \sum_{n=0}^{\infty} 3a_n (\lambda+n)(\lambda+n-1) x^{n-1} + \sum_{n=0}^{\infty} a_n (\lambda+n) x^{n-1} - \sum_{n=0}^{\infty} a_n x^n \right\}$$

$$x^{\lambda} \Big[3a_0 \lambda(\lambda-1) x^{-1} + a_0 \lambda x^{-1} + \sum_{n=1}^{\infty} 3a_n (\lambda+n)(\lambda+n-1) x^{n-1}$$
$$+ \sum_{n=1}^{\infty} a_n (\lambda+n) x^{n-1} - \sum_{n=0}^{\infty} a_n x^n \Big]$$

$$x^{\lambda} \Big\{ a_0 x^{-1} \big(3\lambda(\lambda-1) + \lambda\big) + \sum_{n=1}^{\infty} 3a_n (\lambda+n)(\lambda+n-1) x^{n-1}$$
$$+ \sum_{n=1}^{\infty} a_n (\lambda+n) x^{n-1} - \sum_{n=0}^{\infty} a_n x^n \Big\}$$

let $\theta = n-1$ $\quad n = \theta + 1$ $\quad n = 1 \Rightarrow \theta = 0$

$n = \infty \Rightarrow \theta = \infty$

$$x^{\lambda}\Big\{a_0x^{-1}\big(3\lambda(\lambda-1)+\lambda\big)+\sum_{n=1}^{\infty}3a_n(\lambda+n)(\lambda+n-1)x^{n-1}$$
$$+\sum_{n=1}^{\infty}a_n(\lambda+n)x^{n-1}-\sum_{n=0}^{\infty}a_nx^n\Big\}$$

let $\theta=n-1$ $\quad n=\theta+1$ $\quad n=1\Rightarrow\theta=0$, $n=\infty\Rightarrow\theta=\infty$

$$x^{\lambda}\Big\{a_0x^{-1}\big[3\lambda^2-2\lambda\big]+\sum_{\theta=0}^{\infty}3a_{\theta+1}(\lambda+\theta+1)(\lambda+\theta)x^{\theta}$$
$$+\sum_{\theta=0}^{\infty}a_{\theta+1}(\lambda+\theta+1)x^{\theta}-\sum_{n=0}^{\infty}a_nx^n\Big\}$$

let $\theta=n$

$$x^{\lambda}\Big\{a_0x^{-1}\underbrace{(3\lambda^2-2\lambda)}_{\text{set}=0}+\sum_{n=0}^{\infty}\big[3a_{n+1}(\lambda+n+1)(\lambda+n)+a_{n+1}(\lambda+n+1)-a_n\big]x^n\Big\}$$

$$3a_{n+1}(\lambda+n+1)(\lambda+n)+a_{n+1}(\lambda+n+1)-a_n=0$$

$$a_{n+1}\big[3(\lambda+n+1)(\lambda+n)+(\lambda+n+1)\big]=a_n$$

$$a_{n+1}=\frac{a_n}{(\lambda+n+1)\big[3\lambda+3n+1\big]}$$

*recurrence relation

$3\lambda^2-2\lambda=0$ indicial equation

$\lambda(3\lambda-2)=0$

$\Rightarrow\lambda=0,\frac{2}{3}$

For $\lambda=\frac{2}{3}$: $\quad a_{n+1}=\dfrac{a_n}{\left(\frac{5}{3}+n\right)(3+3n)}$

$$=\frac{a_n}{\quad}$$

May 2-9:36 AM

For $\lambda = \frac{2}{3}$:

$$a_{n+1} = \frac{a_n}{(5+3n)(1+n)}$$

let $a_0 = 1$

for $n=0$: $a_1 = \frac{a_0}{5} = \frac{1}{5}$

$n=1$: $a_2 = \frac{a_1}{16} = \frac{1/5}{16} = \frac{1}{16 \cdot 5}$

$n=2$: $a_3 = \frac{a_2}{33} = \frac{1/16 \cdot 5}{33} = \frac{1}{33 \cdot 16 \cdot 5}$

$\vdots$

$$\Rightarrow y_1(x) = 1 + \frac{1}{5}x + \frac{1}{16 \cdot 5}x^2 + \frac{1}{33 \cdot 16 \cdot 5}x^3 + \cdots$$

$$a_{n+1} = \frac{a_n}{(n+\lambda+1)(3\lambda+3n+1)}$$

For $\lambda=0$: $$a_{n+1} = \frac{a_n}{(n+1)(3n+1)}$$

let $a_0 = 1$

for $n=0$: $a_1 = \frac{a_0}{1} = 1$

$n=1$: $a_2 = \frac{a_1}{8} = \frac{1}{8}$

$n=2$: $a_3 = \frac{a_2}{21} = \frac{1}{21\cdot 8}$

$n=3$: $a_4 = \frac{a_3}{40} = \frac{1}{40\cdot 21\cdot 8}$

$$y_2(x) = 1 + x + \frac{1}{8}x^2 + \frac{1}{21\cdot 8}x^3 + \frac{1}{40\cdot 21\cdot 8}x^4 + \cdots$$

$\Rightarrow$ The general solution is then:

$$y(x) = c_1 y_1(x) + c_2 y_2(x)$$

$$y(x) = c_1\left[1 + \frac{1}{5}x + \frac{1}{16\cdot 5}x^2 + \frac{1}{33\cdot 16\cdot 5}x^3 + \cdots\right]$$
$$+ c_2\left[1 + x + \frac{1}{8}x^2 + \frac{1}{21\cdot 8}x^3 + \frac{1}{40\cdot 21\cdot 8}x^4 + \cdots\right]$$

May 3-9:40 AM