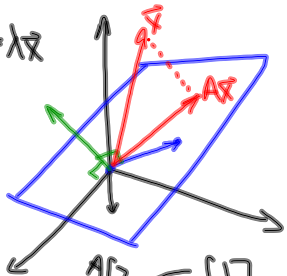
$3x - y + 2z = 0$ is a plane through the origin in 3-space.

a) Find a nonzero point in the plane and write it as a vector.

b) Find the dot product of your answer to (a) and the vector $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$. What do you notice?

$ax + by + cz = d$ $\quad \vec{n} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$A\vec{x} = \lambda \vec{x}$



$A\vec{x} =$

$A[\;] = \vec{0}$

$\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ is an eigenvector of A with $\lambda = 0$

$A[\;] = [\;]$

$\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ is an eigenvector with eigenvalue $\lambda = 1$

$\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue $1 - \lambda$ $\quad A = PDP^{-1}$

$$P = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & -1 \\ -1 & 1 & 2 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$PDP^{-1} = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & -1 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} P^{-1}$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \frac{11}{28} & \frac{1}{28} & -\frac{16}{28} \\ -\frac{1}{28} & \frac{5}{28} & \frac{4}{28} \\ \frac{6}{28} & -\frac{3}{28} & \frac{4}{28} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{10}{28} & \frac{6}{28} & -\frac{12}{28} \\ \frac{6}{28} & \frac{26}{28} & \frac{4}{28} \\ -\frac{12}{28} & \frac{4}{28} & \frac{20}{28} \end{bmatrix} = \frac{1}{14} \begin{bmatrix} 5 & 3 & -6 \\ 3 & 13 & 2 \\ -6 & 2 & 10 \end{bmatrix} = A$$

$\vec{u} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ $\quad A\vec{u} = \begin{bmatrix} \frac{9}{7} \\ \frac{39}{7} \\ \frac{6}{7} \end{bmatrix}$

$3x - y + 2z = 0$

$3\left(\frac{9}{7}\right) - \left(\frac{39}{7}\right) + 2\left(\frac{6}{7}\right) \stackrel{?}{=} 0$

$0 = 0$

$$\vec{x}'' = \begin{bmatrix} -3 & 1 \\ 2 & -2 \end{bmatrix} \vec{x}, \quad \vec{x}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$x'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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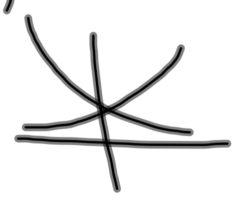
$$\delta_h(t) = \begin{cases} \frac{1}{2h} & \text{if } -h<t<h \\ 0 & \text{elsewhere} \end{cases}$$

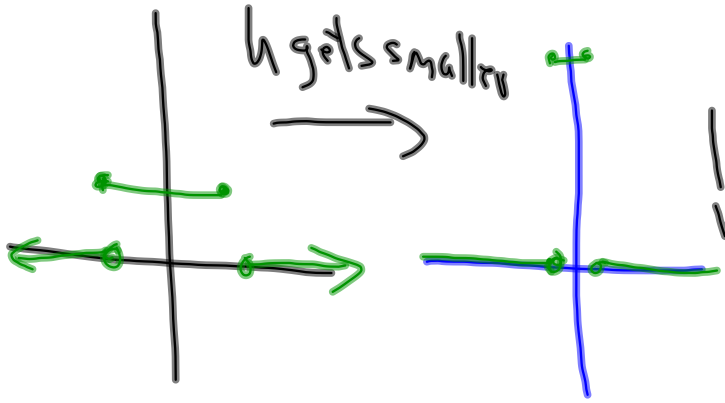
$h>0$ fixed

$y = ax^2 + bx + c$

3-parameter family

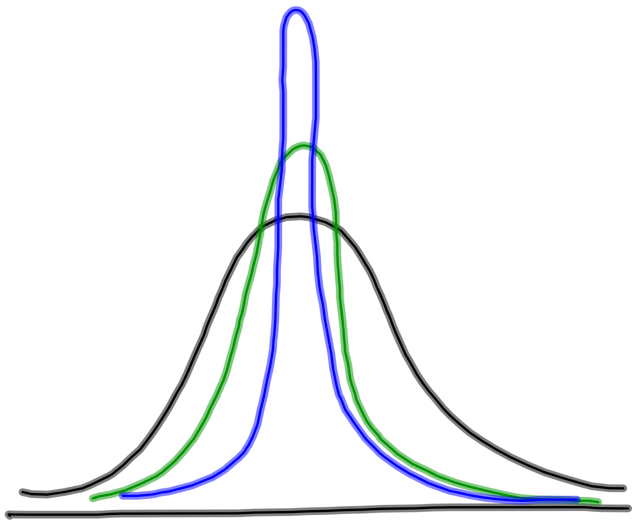
$y = Ce^{kt}$





$$\lim_{h\to 0} \int_0^\infty f(t)\,\delta_h(t-c)\,dt = f(c)$$

$$f(t)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{t}{\sigma}\right)^2}$$



$$\delta_h(t) \xrightarrow[h \to 0]{} \delta(t)$$

"delta function"

$$\delta(t) = \begin{cases} ? & \text{if } t=0 \\ 0 & \text{elsewhere} \end{cases}$$

Laurent Schwarz

distributions

$$\int_0^\infty f(t)\,\delta(t-c)\,dt = f(c)$$

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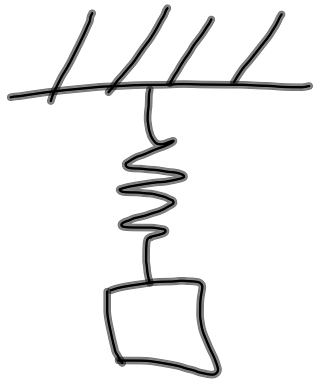
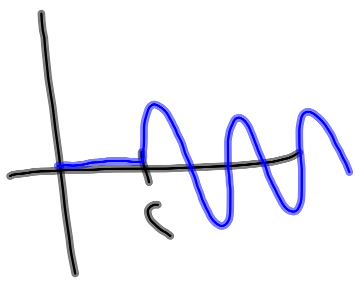
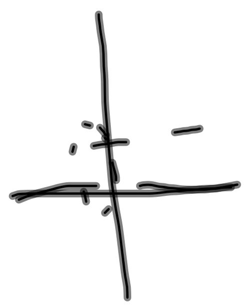


$\delta(t-c)$ applies an impulse to the system at time $t=c$.



$$\mathcal{L}\{\delta(t-c)\} = \int_0^\infty \delta(t-c)e^{-st}\,dt = e^{-cs}$$



$$\mathcal{L}\{\delta(t)\} = \lim_{c\to 0}\mathcal{L}\{\delta(t-c)\} = \lim_{c\to 0} e^{-cs} = 1$$

$$\delta(t-c) \longleftrightarrow e^{-cs}$$

$$\delta(t) \longleftrightarrow 1$$