Assignment 11

Don't try to get factorials!

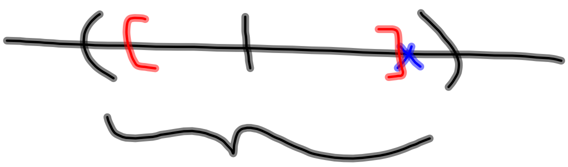
1/2(b) solution:

$$y = a_0\left(1 + 2x^2 + 3x^4 + 4x^5 + \cdots\right) + a_1\left(x + \frac{5}{3}x^3 + \frac{7}{3}x^5 + \frac{9}{3}x^7 + \cdots\right)$$

1/2(d) solution

$$y = a_0\left(1 + \frac{\alpha^2}{4\cdot 3}x^4 + \frac{\alpha^4}{8\cdot 7\cdot 4\cdot 3}x^8 + \frac{\alpha^6}{12\cdot 11\cdot 8\cdot 7\cdot 4\cdot 3}x^{12} + \cdots\right) + a_1\left(x + \frac{\alpha^2}{5\cdot 4}x^5 + \frac{\alpha^4}{9\cdot 8\cdot 5\cdot 4}x^9 + \frac{\alpha^6}{13\cdot 12\cdot 9\cdot 8\cdot 5\cdot 4}x^{13} + \cdots\right)$$

Assignment 12



Interval of convergence

$(\; , \;)$ not $[\; , \;]$

$y = a_0 + a_1 x + a_2 x^2 + \cdots$

May 6-9:03 AM

⇒ The general solution is then:

$$y(x) = c_1 y_1(x) + c_2 y_2(x)$$

$\lambda = \frac{2}{3}$

$$y(x) = c_1\left[1 + \tfrac{1}{5}x + \tfrac{1}{16\cdot 5}x^2 + \tfrac{1}{33\cdot 16\cdot 5}x^3 + \cdots\right]$$

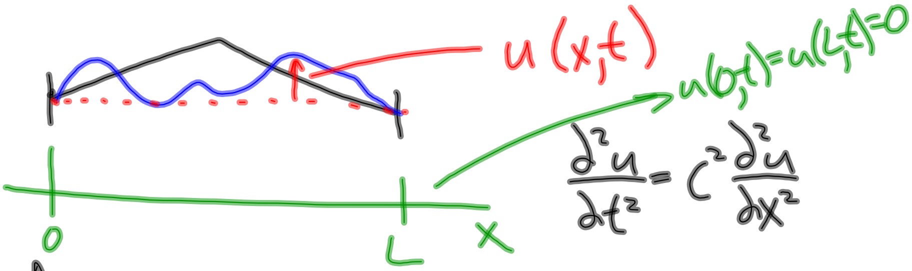
$\lambda = 0$

$$+ c_2\left[1 + x + \tfrac{1}{8}x^2 + \tfrac{1}{21\cdot 8}x^3 + \tfrac{1}{40\cdot 21\cdot 8}x^4 + \cdots\right]$$

$x^{\frac{2}{3}}$

$1 = x^0$

Assumption: $y = x^{\lambda}\sum_{n=0}^{\infty} a_n x^n$



$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

Assume $u(x,t) = \phi(x)h(t)$

$$\frac{d^2h}{dt^2} = -\lambda c h \qquad \frac{d^2\phi}{dx^2} = -\lambda\phi$$

$\phi = e^x$

$\phi' = -e^x$

$\phi'' = -e^x$

$= \phi$

$$\frac{d^2}{dx^2}(\phi) = -\lambda\phi$$

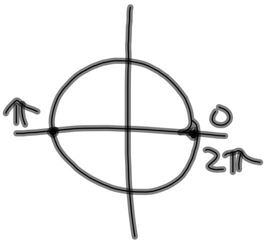
$$A\vec{x} = \lambda\vec{x}$$

$$\phi(x) = A\sin\sqrt{\lambda}x + B\cos\sqrt{\lambda}x$$

$$\phi(0) = 0 = B\cos\sqrt{\lambda}x \quad B=0$$

$$\boxed{\phi(x) = A\sin\sqrt{\lambda}x} \qquad u(L,t) = 0$$

$$\phi(L) = A\sin\sqrt{\lambda}L = 0$$

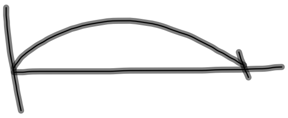


$$\sqrt{\lambda}L = n\pi$$

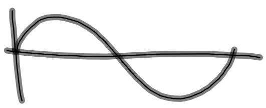
$$\sqrt{\lambda} = \frac{n\pi}{L}$$

$$\phi(x) = A\sin\frac{n\pi}{L}x$$

n=1:



n=2:



$$u(x,t) = \sum_{n=1}^{\infty} \underbrace{\left(A\sin\frac{n\pi c}{L}t + B\cos\frac{n\pi c}{L}t\right)}_{h(t)} \underbrace{\sin\frac{n\pi}{L}x}_{\phi(x)}$$

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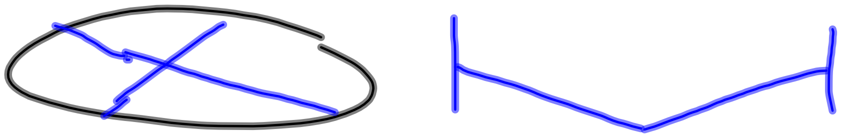
Vibrating circular membrane



$$u = u(r,t) = \phi(r)h(t)$$

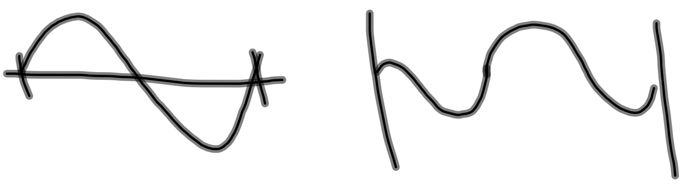
$$r^2\frac{d^2\phi}{dr^2} + r\frac{d\phi}{dr} + r^2\phi = 0$$

$$\frac{d^2h}{dt^2} = -\lambda^2c^2h$$

Hit it w/ MOF!

$$\phi(r) = c_1 J_0(kr) + \cancel{c_2 Y_0(kr)}$$

$$u(r,t) = \left(A_n \sin c\lambda_n t + B_n \cos c\lambda_n t\right) J_0(\lambda_n r)$$



$$u(r,\theta,t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} u_{mn}(r,\theta,t)$$

$$u_{mn} = (A_n \sin\lambda_n t + B_n \cos\lambda_n t)\, J_m(\lambda_{mn} r)\,(C_m \sin m\theta + D_m \cos m\theta)$$

Gives u for a fixed time $\leftarrow$