Learning math:

* techniques
* applications
* intuition
* validation (proof)

This course

2nd order

$a(x)y'' + b(x)y' + c(x)y = f(x)$

* series solutions — linear, non-constant coefficien DEs.

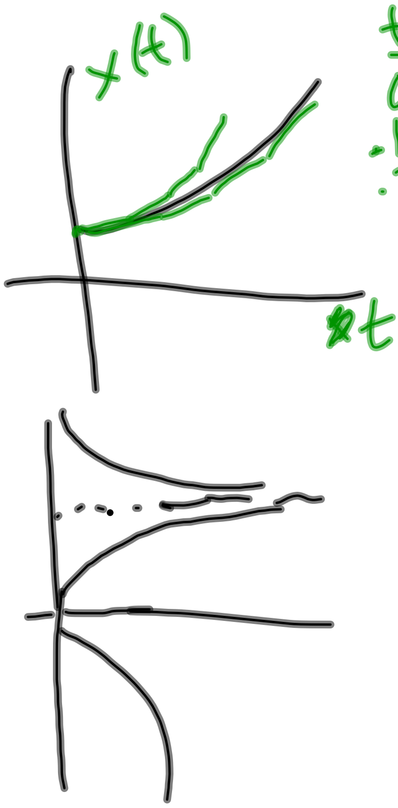
  Bessel funtions

* Laplace transforms — linear, constant coeffient, discontinuous or impulse forcing

* systems — multiple related variables, higher order ODEs

Studying DEs - methods

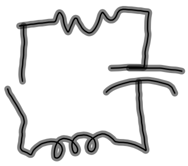
* analytical
* numerical
* qualitative



t	x
0	1
.1	
.2	

$$\vec{x}' = A\vec{x} \qquad \vec{x} = \vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \vec{x}$$



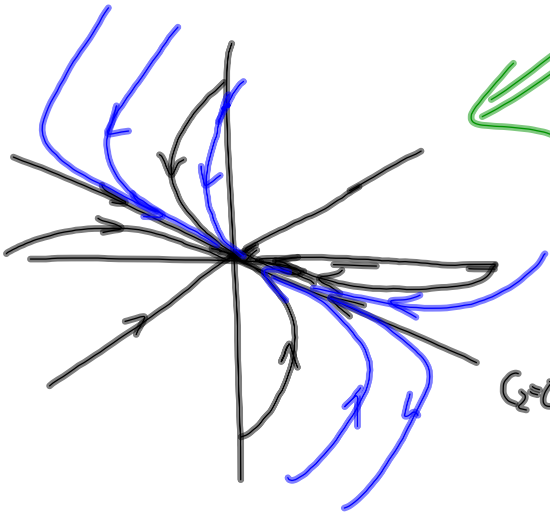
$$\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 3x_1 + 4x_2 \end{aligned} \Longrightarrow \text{sol } \vec{x} = \quad \Longrightarrow \begin{aligned} x_1 &= \\ x_2 &= \end{aligned}$$

Phase plane:

t	x_1	x_2
0	.	.
⋮	⋮	⋮

plot

With enough trajectories this is a phase plot/portrait.



$$x_1 = c_1 2e^{-3t} + c_2 2e^{-t}$$

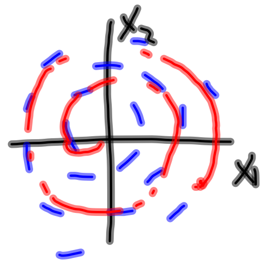
$$x_2 = c_1 e^{-3t} + c_2 e^{-t}$$

scalar

$$c_2 = 0: \ \vec{x} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-3t}$$

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \vec{x}$$

Direction field



$$\frac{dx_2}{dx_1}$$

Creating direction field

At $(2,1)$:

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix} = \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix}$$

$$\left.\frac{dx_1}{dt}\right|_{(2,1)} = 4 \quad , \quad \frac{dx_2}{dt} = 10$$

$$\frac{dx_2}{dx_1} = \frac{dx_2/dt}{dx_1/dt} = \frac{10}{4} = \frac{5}{2}$$