We have $H(s) = F(s)G(s)$

$\left(\text{Ex: } H(s) = \frac{1}{s^2(s-3)} = \frac{1}{s^2} \cdot \frac{1}{s-3} \right)$

What does $h(t)$ look like?

$$f(t) \longleftrightarrow F(s) = \int_0^\infty f(u)e^{-su}\,du$$

$$g(t) \longleftrightarrow G(s) = \int_0^\infty g(w)e^{-sw}\,dw$$

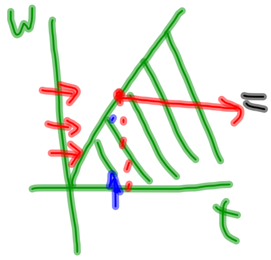
$$H(s)=F(s)G(s)=\left(\int_0^\infty f(u)e^{-su}\,du\right)\left(\int_0^\infty g(w)e^{-sw}\,dw\right)$$

Let $t=u+w$

$dt=du$

$u=t-w$

$$=\int_{w=0}^{\infty}\int_{u=0}^{\infty} f(u)g(w)e^{-s(u+w)}\,du\,dw$$



$$=\int_{w=0}^{\infty}\int_{t=w}^{\infty} f(t-w)g(w)e^{-st}\,dt\,dw$$

τ tau

$$=\int_{t=0}^{\infty}\int_{w=0}^{t} f(t-w)g(w)e^{-st}\,dw\,dt$$

or

$$=\int_{t=0}^{\infty}\left(\int_{\tau=0}^{t} f(t-\tau)g(\tau)\,d\tau\right)e^{-st}\,dt$$

$$=\mathcal{L}\{h(t)\}$$

where $h(t)=\int_{\tau=0}^{t} f(t-\tau)g(\tau)\,d\tau$

$$h(t)=(f*g)(t)=\int_{\tau}^{t} f(t-\tau)g(\tau)\,d\tau,$$

the convolution of f and g.

Facts: $f*g=g*f$

$$\mathcal{L}^{-1}\{F(s)G(s)\}=(f*g)(t)$$

$$\mathcal{L}\{(f*g)(t)\}=F(s)G(s)$$

$$x^a x^b = x^{a+b}$$

$$\log(ab)=\log a+\log b$$

Applying $\mathcal{L}\{f*g\} = FG$

$$\mathcal{L}\left\{ \overbrace{\int_0^t \underbrace{3(t-\tau)^2}_{f(t-\tau)} \underbrace{\sin 2\tau}_{g(\tau)} \, d\tau}^{(f*g)(t)} \right\}$$

$$= \mathcal{L}\{3t^2 * \sin 2t\}$$

$$= \mathcal{L}\{3t^2\} \cdot \mathcal{L}\{\sin 2t\}$$

$$= 3 \cdot \frac{2}{s^3} \cdot \frac{2}{s^2+4} = \frac{12}{s^3(s^2+4)}$$

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Use convolution rule for inverse transform. Give result as a convolution.

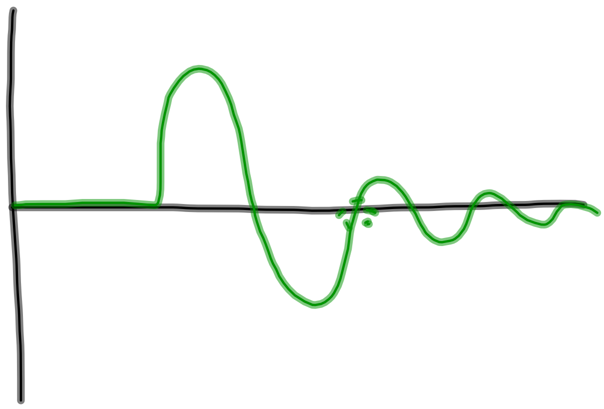
$$F(s) = \frac{3s}{(s^2+16)^2} = \underbrace{\overset{F(s)}{\frac{3}{(s^2+16)}}}_{\mathcal{L}\{\underbrace{\frac{3}{4}\sin 4t}_{f(t)}\}} \cdot \underbrace{\overset{G(s)}{\frac{s}{(s^2+16)}}}_{\mathcal{L}\{\underbrace{\cos 4t}_{g(t)}\}} \qquad f(t) = ?$$

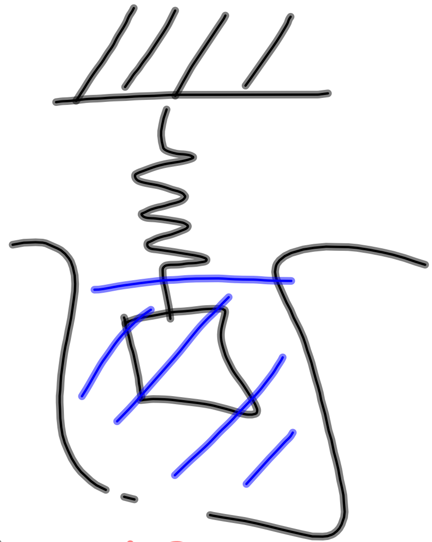
$$f(t) = \int_0^t \frac{3}{4}\sin 4(t-\tau)\cos 4\tau \, d\tau$$

You try $G(s) = \dfrac{1}{s^2(s+3)}$

$$G(s) = \frac{1}{s^2(s+3)} = \underset{\mathcal{L}\{t\} \to f(t)}{\overset{F(s)}{\frac{1}{s^2}}} \cdot \underset{\mathcal{L}\{e^{-3t}\} \to g(t)}{\overset{G(s)}{\frac{1}{s+3}}}$$

$$g(t) = \int_0^t (t-\tau)e^{-3\tau}\,d\tau$$





$$y'' + 4y' + 13y = \delta(t-4) + A\delta(t-c)$$

$$y = \frac{1}{3} e^{-2(t-4)} \sin\left[3(t-4)\right] u(t-4)$$

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