$$\frac{d}{dx}\left[f(x)+g(x)\right] = f'(x)+g'(x)$$

$$\frac{d}{dx}\left[f(x)g(x)\right] \neq f'(x)g'(x)$$

$$= f(x)g'(x)+g(x)f'(x)$$

$$\mathcal{L}\{f(t)g(t)\} \neq \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{\underline{f(t) \overset{?}{*} g(t)}\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$$

$$\int_0^t f(t-\tau)g(\tau)\,d\tau = F(s)G(s)$$

$\Uparrow$

$$ay'' + by' + cy = g(t) \quad y(0) = c_1, \; y'(0) = c_2$$

$$a\left(s^2 Y(s) - sc_1 - c_2\right) + b\left(sY(s) - c_1\right) + cY(s) = G(s)$$

$$\left(as^2 + bs + c\right)Y(s) - asc_1 - ac_2 - bc_1 = G(s)$$

$$\left(as^2 + bs + c\right)Y(s) = (as+b)c_1 + ac_2 + G(s)$$

$$Y(s) = \underbrace{\frac{(as+b)c_1 + ac_2}{as^2+bs+c}}_{\Phi(s)} + \underbrace{\frac{G(s)}{as^2+bs+c}}_{\Psi(s)}$$

$$y(t) = \phi(t) + \psi(t) \qquad + \underbrace{\frac{1}{as^2+bs+c}}_{H(s) \text{ transfer function}} \cdot G(s)$$

$$\psi(t) = h(t) * g(t)$$

$$ay''+by'+cy=\delta(t) \quad y(0)=0, y'(0)=0$$

$$as^2Y(s)+bsY(s)+cY(s)=1$$

$$Y(s)=\frac{1}{as^2+bs+c}=H(s) \text{ transfer function}$$

$h(t)$ is the impulse response of the system.

$$\text{Input} \longrightarrow \boxed{\text{System}} \longrightarrow \text{output (response)}$$

If $g(t)$ goes in, what comes out?

How to answer:

Send in $\delta(t)$, look at what comes out. It is $h(t)$.

Answer: $$\underbrace{\int_0^t \underbrace{h(t-\tau)}_{\text{measured}}\,\underbrace{g(\tau)}_{\text{known}}\,d\tau}_{\text{Done numerically}}$$