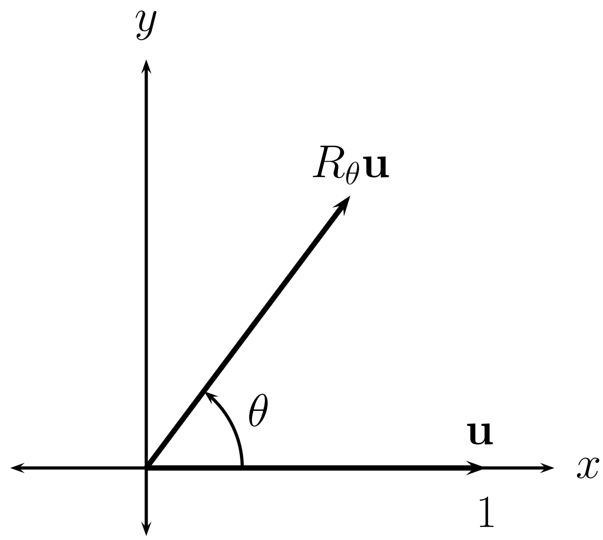
1. To the right we see the vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $R_\theta \mathbf{u}$, the result when a rotation of θ is applied to $\mathbf{u}$.



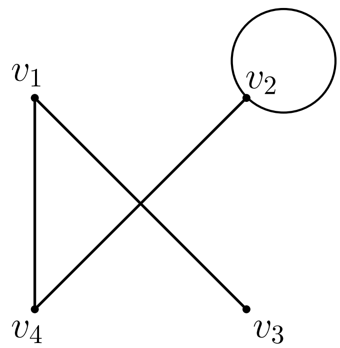
   (a) What is $\|R_\theta \mathbf{u}\|$, the magnitude (length) of $R_\theta \mathbf{u}$?

   (b) Determine the components of $R_\theta \mathbf{u}$.

   (c) Suppose that R_θ is accomplished by the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Apply the matrix to $\mathbf{u}$ and set the result equal to your answer to (b) in order to determine some of the entries in the matrix.

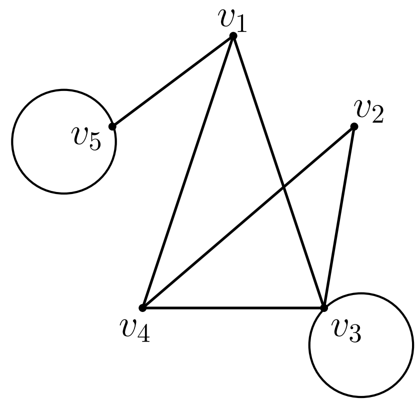
   (d) Apply the same process to the vector $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ to obtain the other entries in the matrix. Give the final matrix for R_θ.

2. The Graph A below has incidence matrix A. What do you think the incidence matrix B of Graph B is?



Graph A

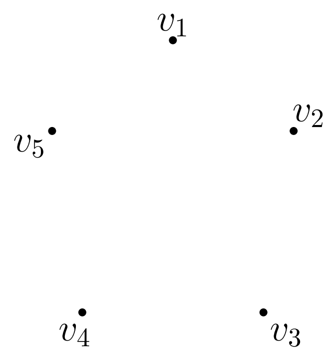
$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$



Graph B

3. Give Graph C represented by the matrix C below for the vertices shown to the right below.

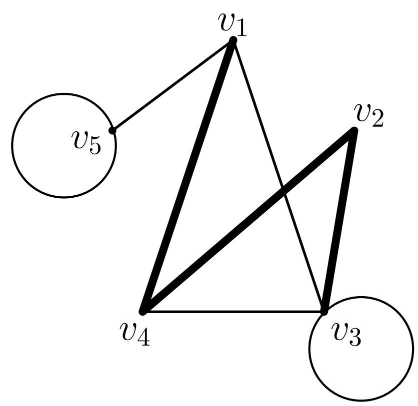
$$C = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Graph C

4. Give a 3×3 matrix of ones and zeros that could *NOT* be an incidence matrix.

5. The darkened edges on the graph to the right are what we call a **3-path** from v_1 to v_3, because three edges are travelled in getting from v_1 to v_3. That particular path is denoted by $v_1v_4v_2v_3$. Find all the 3-paths you can from v_1 to v_3. *We are allowed to travel the same edge more than once.*



Graph B

6. Find the number of 2-paths from v_1 to v_2, and the number of 2-paths from v_3 to itself.

7. For the incidence matrix B for Graph B, calculate B^2 and look at the $(1, 2)$ and $(3, 3)$ entries. What do you notice?

8. How do you think we can determine the number of 3-paths from v_1 to v_3. Do that and see if it agrees with the number you found.

9. The graph to the right is called a **directed graph** or, by people "in the know" (like you), a **digraph**. What do you think its incidence matrix is? What characteristic of matrices for "regular" graphs is this matrix lacking?