

Put all work and answers on additional paper.

1. For each of the following matrices B and C and the given vectors, determine whether any of the vectors are eigenvectors of the matrix, **showing clearly how you do it**. For those that are, say so and give the corresponding eigenvalues. **Do so in a sentence of the form**

\mathbf{u} is an eigenvector of A with corresponding eigenvalue of *number*.

$$(a) \quad A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$(b) \quad B = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 5 \\ -15 \\ -5 \end{bmatrix}$$

$$(c) \quad C = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

2. In this exercise you will find the eigenvalues and corresponding eigenspaces of the matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.

- Follow the process of Example 5.6(a) to find the eigenvalues.
- Choose one of your eigenvalues (call it λ_1) from (a) and follow the process of Example 5.6(b) to find the eigenspace E_{λ_1} corresponding to that eigenvalue.
- The eigenspace is the set of all scalar multiples of some vector. Multiply A times that vector to see if the result is λ_1 times that vector. If it isn't you have an error somewhere - find it.
- Repeat parts (b) and (c) for the other eigenvalue λ_2 .

3. Find the eigenvalues and corresponding eigenspaces for the matrix $B = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$.