

$$\textcircled{1} \text{ a) } A = \begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}, \vec{u} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A\vec{u} = \begin{bmatrix} -12 \\ 3 \end{bmatrix} = 3\vec{u} \quad \vec{u} \text{ is an eigenvector of } A \text{ with eigenvalue } 3$$

$$A\vec{v} = \begin{bmatrix} -14 \\ -2 \end{bmatrix} \quad \vec{v} \text{ is not an eigenvector}$$

$$A\vec{w} = \begin{bmatrix} -2 \\ -2 \end{bmatrix} = -2\vec{w} \quad \vec{w} \text{ is an eigenvector of } A \text{ with eigenvalue } -2$$

$$\text{b) } B = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{bmatrix}, \vec{u} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \vec{v} = \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \vec{w} = \begin{bmatrix} 5 \\ -15 \\ -5 \end{bmatrix}$$

$$B\vec{u} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0\vec{u} \quad \vec{u} \text{ is an eigenvector of } B \text{ with eigenvalue } 0$$

$$B\vec{v} = \begin{bmatrix} 2 \\ -6 \\ -2 \end{bmatrix} = -2\vec{v} \quad \vec{v} \text{ is an eigenvector of } B \text{ with eigenvalue } -2$$

$$B\vec{w} = \begin{bmatrix} -10 \\ 30 \\ 10 \end{bmatrix} = -2\vec{w} \quad \vec{w} \text{ is an eigenvector of } B \text{ with eigenvalue } -2$$

$$\text{c) } C = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}, \vec{u} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \vec{v} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$C\vec{u} = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} = \vec{u} \quad \vec{u} \text{ is an eigenvector of } C \text{ with eigenvalue } 1$$

$$C\vec{v} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} = -3\vec{v} \quad \vec{v} \text{ is an eigenvector of } C \text{ with eigenvalue } -3$$

$$C\vec{w} = \begin{bmatrix} 6 \\ 0 \\ -3 \end{bmatrix} = -3\vec{w} \quad \vec{w} \text{ is an eigenvector of } C \text{ with eigenvalue } -3$$

$$\textcircled{2} \quad A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$$

$$a) \quad A - \lambda I = \begin{bmatrix} 1-\lambda & 3 \\ 4 & 2-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda)(2-\lambda) - 12 = 2 - 3\lambda + \lambda^2 - 12 = \lambda^2 - 3\lambda - 10 = (\lambda - 5)(\lambda + 2)$$

$$\lambda = -2, 5$$

$$b) \quad \lambda_1 = -2: \quad A - \lambda_1 I = \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \quad \begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$c) \quad \lambda_2 = 5: \quad A - \lambda_2 I = \begin{bmatrix} -4 & 3 \\ 4 & -3 \end{bmatrix} \quad \begin{bmatrix} -4 & 3 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \vec{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$E_1 = \left\{ t \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \quad E_2 = \left\{ t \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right\}$$

$$\textcircled{3} \quad B = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \quad B - \lambda I = \begin{bmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{bmatrix} \quad \det(B - \lambda I) = (1-\lambda)(4-\lambda) + 2$$

$$= \lambda^2 - 5\lambda + 6$$

$$= (\lambda - 2)(\lambda - 3)$$

$$\lambda = 2, 3$$

$$\lambda = 2: \quad \begin{bmatrix} -1 & -2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad E_1 = \left\{ t \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$$

$$\lambda = 3: \quad \begin{bmatrix} -2 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad E_2 = \left\{ t \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$