Math 341 ASSIGNMENT 14, WINTER 2017 **Due at the start of class on Wed, March 15th**

Put all work and answers on additional paper.

1. (a) Find the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$.

   (b) Let P be the 2×2 matrix whose columns are the two eigenvectors. Find the inverse matrix P^{-1} using the formula $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \implies A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$. **Leave the fraction** $\dfrac{1}{ad-bc}$ **out of the matrix**.

   (c) Compute $D = P^{-1}AP$, not multiplying the fraction in until the very end. What kind of a matrix is the result? (The letter used for it is a hint!) What do you notice about its entries?

2. Repeat the process you carried out in the previous exercise for $B = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ to find the matrices P and D. *This time you should not need to do a multiplication to find D!*

In the above two exercises you started with a matrix A and found matrices P and D based on the eigenvectors and eigenvalues of A. For the next exercise (on the back) you will do the opposite: You will determine two eigenvectors and their corresponding eigenvalues to come up with the matrices P and D, from which you will compute A,

3. The goal of this exercise is to use eigenvectors and eigenvalues to find a matrix B that reflects all vectors across the line l in $\mathbb{R}^2$ through the origin and the point $(3, 2)$.

   (a) Sketch the coordinate axes and the line.

   (b) Give a vector $\mathbf{u}$ on the line l and find $B\mathbf{u}$. (No, you don't know B yet, but you should know what $B\mathbf{u}$ is, based on understanding how reflections behave.) This indicates that $\mathbf{u}$ is an eigenvector of B. What is the corresponding eigenvalue?

   (c) Give a vector $\mathbf{v}$ perpendicular to the line l and find $B\mathbf{v}$. This indicates that $\mathbf{v}$ is an eigenvector of B. What is the corresponding eigenvalue?

   (d) Give the matrices P and D. Find P^{-1}, and the compute B by $B = PDP^{-1}$.

   (e) Plot the vector $\mathbf{w} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ accurately on your graph. Test your B by finding $B\mathbf{w}$ and making sure it looks the way you expected it to, knowing that B is a reflection across l.