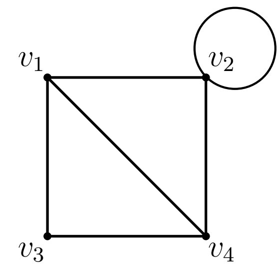
Math 341 ASSIGNMENT 6, WINTER 2017 **Due at the start of class on Monday, February 13th**

1. Use the graph to the right for the following.

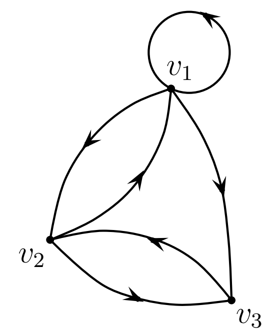


   (a) Give the incidence matrix for the graph; call it A. Find and give A^3 also.

   (b) How many 3-paths from v_1 to v_4 do you expect? Give all of them by listing the vertices the path goes through, *in order and including* v_2 *and* v_4, as done in class. (For example, then, a path from v_4 to v_3, through v_2 would be denoted $v_4v_2v_3$.)

   (c) How many 3-paths are there from v_4 to v_1? What characteristic of the matrix A^3 relates your answer to the number of 3-paths from v_1 to v_4?

2. Use the directed graph to the right for the following.



   (a) Give the incidence matrix; again, call it A.

   (b) The number of n-paths from vertex i to vertex j is given by the (i, j) entry of A^n. How many 4-paths are there from v_1 to v_3? Show how you get your answer.

   (c) Give all 4-paths from v_2 to v_3.

   (d) Give all 3-paths from v_3 to v_2. Is it the same as the number from v_2 to v_3?

Some Trig Identities

You will find the following useful in understanding/answering the following exercise.

$$\sin^2\theta + \cos^2\theta = 1 \qquad \cos(-\theta) = \cos\theta \qquad \sin(-\theta) = -\sin\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$$

3. Consider the general rotation matrix $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

   (a) Suppose that we were to apply A to a vector $\mathbf{x}$, then apply A again, to the result. Thinking only geometrically (don't do any calculations), give a single matrix B that should have the same effect.

   (b) Find the matrix A^2 algebraically, by multiplying A by itself.

   (c) Use some of the trigonometric facts above to continue your calculations from part (b) until you arrive at matrix B. This of course shows that that $B = A^2$.

There is another exercise on the back!

4. The point of this is to determine a reflection matrix $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ that reflects all vectors across the line with equation $y = \frac{3}{4}x$.

(a) Sketch the line accurately and then give a vector $\mathbf{u}$ (label it as such on your paper!) that is on the line, and a vector $\mathbf{v}$ that is perpendicular to the line.

(b) Tell what $C\mathbf{u}$ and $C\mathbf{v}$ should be, labeling each as such.

(c) Multiply $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ by your $\mathbf{u}$ and set the result equal to what you said $C\mathbf{u}$ should be. This gives you two linear equations, one in a and b, the other in c and d. Give them both clearly.

(d) Repeat (b) for the vector $\mathbf{v}$.

(e) Give the system of two equations for a and b, and solve for a and b with your calculator or an online tool that will give exact answers. **Give a and b in exact (fraction) form.**

(f) Give the system of equations in c and d and its exact solution, again using technology to find it.

(g) Give the matrix C and test it by letting it act on a few vectors, using the *UCSMP Polygon Plotter*. If it doesn't work as advertised, find and correct your error!

(h) **Challenge:** Note that the line $y = -\frac{3}{2}x$ goes through the origin and the point $(-2, 3)$. Look for patterns in your matrix C involving those two numbers, and then try giving a matrix C that will reflect vectors across a line through the origin and an arbitrary point (a, b).