

Show all work on another sheet of paper, except the graphing that you do on the back side.

- Plot the points  $(1, 0)$ ,  $(2, 2)$ ,  $(3, 2)$ ,  $(4, 3)$ ,  $(5, 5)$ ,  $(6, 6)$  on the grid below. It should be clear from the graph that there is no line containing all of the points. The objective of this exercise is to find the line that “best fits” all of the points.
- Substitute the first and last points into  $y = a_1x + a_0$  and give the resulting system of equations. Then write the system in matrix form. Note that the vector  $\mathbf{x}$  is  $\mathbf{x} = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$ .
- Solve the system using Cramer’s rule, and then give the equation of the line through  $(1, 1)$  and  $(6, 7)$ . **Using a straightedge**, draw the line in on the graph.
- Now substitute *ALL* of the points into  $y = a_1x + a_0$  and give the resulting matrix equation. Note that in this case the matrix  $A$  is not square.
- Compute the vector  $\bar{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}$ , rounding each component to the hundredth’s place. **I would expect you to do this with your calculator or some other tool if you can.** This vector  $\bar{x}$  is the best (in a sense we’ll discuss later) we can do at finding a solution  $\mathbf{x}$  to  $A\mathbf{x} = \mathbf{b}$ .
- Give the equation  $y = a_1x + a_0$  of the line that results from your values obtained in Exercise 7. *If the slope and y-intercept are not close to those you got in Exercise 3, something is wrong. Try to find and fix your error.*
- Even though  $A\bar{\mathbf{x}}$  is not equal to  $\mathbf{b}$ , we can see how close it is by computing the **least squares error vector**  $\mathbf{b} - A\bar{\mathbf{x}}$ . Compute it, with each component to the nearest hundredth.
- Find the **least squares error**, which is the magnitude  $\|\mathbf{b} - A\bar{\mathbf{x}}\|$  of the least squares error vector.

