**Math 341** ASSIGNMENT 9, WINTER 2017 **Due at the start of class on Monday, February 27th**

1. In the blank next to each set $\mathcal{S}$, tell whether the span of the set is

   (a) a point (b) a line (c) a plane (d) all of $\mathbb{R}^3$

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\} \quad \_\_\_\_\_ \qquad \mathcal{S}_2 = \left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 3 \\ 0 \end{bmatrix} \right\} \quad \_\_\_\_\_$$

$$\mathcal{S}_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\} \quad \_\_\_\_\_ \qquad \mathcal{S}_4 = \left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 13 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix} \right\} \quad \_\_\_\_\_$$

2. Refer to the sets from the previous exercise. Give a vector meeting each of the following conditions in the appropriate space below. Write *DNE* if no such vector exists.

   (a) A vector in $\mathbb{R}^2$ that *IS NOT* in the span of $\mathcal{S}_1$. (b) A vector in $\mathbb{R}^3$ that *IS NOT* in the span of $\mathcal{S}_2$.

   (c) A non-zero vector in $\mathbb{R}^3$ *other than the one given* that *IS* in the span of $\mathcal{S}_3$.

   (a) (b) (c)

3. Again referring to the sets from Exercise 1, list all that are linearly independent: ______________

4. For the following, **give some indication of how you get your answers**.

   (a) $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix}$ are linearly (circle one) independent dependent

   **If dependent**, give $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$: $\mathbf{v} = \_\_\_\_\_\_\ \mathbf{u} + \_\_\_\_\_\_\ \mathbf{w}$

   (b) $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix}$ are linearly independent dependent

   **If dependent**, give $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$: $\mathbf{v} = \_\_\_\_\_\_\ \mathbf{u} + \_\_\_\_\_\_\ \mathbf{w}$

5. Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$.

   (a) Circle all that are in the null space of $A$: $\mathbf{u}$ $\mathbf{v}$ $\mathbf{w}$

   (b) For any that *ARE* in null$(A)$, give a computation that verifies that.

6. Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$.

(a) Circle all that are in the column space of $A$: $\mathbf{u}$ $\mathbf{v}$ $\mathbf{w}$

(b) For any that *ARE* in $\text{col}(A)$, give an equation that verifies that.

7. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

(a) Give a basis for the null space of $A$.

(b) Give a basis for the column space of $A$.

8. In the blank next to each set $\mathcal{S}$, give the appropriate Roman numeral (one per set), selected from the following.

I. The set *IS* a basis for $\mathbb{R}^3$.

II. The set is not a basis for $\mathbb{R}^3$ because it does not span $\mathbb{R}^3$.

III. The set is not a basis for $\mathbb{R}^3$ because the vectors are not linearly independent.

IV. The set is not a basis for $\mathbb{R}^3$ because it does not span $\mathbb{R}^3$ *AND* the vectors are not linearly independent.

$$\mathcal{S}_1 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\} \quad \underline{\qquad}$$

$$\mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\} \quad \underline{\qquad}$$

$$\mathcal{S}_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad \underline{\qquad}$$

$$\mathcal{S}_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \quad \underline{\qquad}$$