10 points

Math 341 ASSIGNMENT 9, WINTER 2017 **Due at the start of class on Monday, February 27th**

1. In the blank next to each set $\mathcal{S}$, tell whether the span of the set is

(a) a point (b) a line (c) a plane (d) all of $\mathbb{R}^3$

$\mathcal{S}_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ c

$\mathcal{S}_2 = \left\{ \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -9 \\ 3 \\ 0 \end{bmatrix} \right\}$ b

+1

$\mathcal{S}_3 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ b

$\mathcal{S}_4 = \left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 13 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 5 \end{bmatrix} \right\}$ c

2. Refer to the sets from the previous exercise. Give a vector meeting each of the following conditions in the appropriate space below. Write *DNE* if no such vector exists.

(a) A vector in $\mathbb{R}^2$ that *IS NOT* in the span of $\mathcal{S}_1$. (b) A vector in $\mathbb{R}^3$ that *IS NOT* in the span of $\mathcal{S}_2$.

(c) A non-zero vector in $\mathbb{R}^3$ *other than the one given* that *IS* in the span of $\mathcal{S}_3$.

(a) DNE (b) $\begin{bmatrix} * \\ * \\ \neq 0 \end{bmatrix}$ (c) $\begin{bmatrix} a \\ a \\ 0 \end{bmatrix}$

$+1\frac{1}{2}$

+1

3. Again referring to the sets from Exercise 1, list all that are linearly independent: S_1, S_3

4. For the following, **give some indication of how you get your answers.**

(a) $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 4 \\ 0 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix}$ are linearly (circle one) independent dependent

If dependent, give $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$: $\mathbf{v} =$ ______ $\mathbf{u}$ + ______ $\mathbf{w}$

$+1\frac{1}{2}$

(b) $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 3 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ 9 \\ 7 \end{bmatrix}$ are linearly independent dependent

If dependent, give $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$: $\mathbf{v} = -5\ \mathbf{u} + 2\ \mathbf{w}$

5. Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$.

(a) Circle all that are in the null space of A: $\mathbf{u}$ $\mathbf{v}$ $\mathbf{w}$

(b) For any that *ARE* in null(A), give a computation that verifies that.

+1

$A\vec{u} = \vec{0}$

$\begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$.

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$c_1 = 5 + 2t$
$c_2 = t$

(a) Circle all that are in the column space of A: **u** **v** **w**

(b) For any that *ARE* in col(A), give an equation that verifies that.

$$(5+2t)\begin{bmatrix} 3 \\ -2 \end{bmatrix} + t\begin{bmatrix} -6 \\ 4 \end{bmatrix} = \begin{bmatrix} 15 \\ -10 \end{bmatrix}$$

+1

7. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. $\xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(a) Give a basis for the null space of A.

$$\vec{x} = \begin{bmatrix} 2t \\ -t \\ t \end{bmatrix} = t\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$$

(b) Give a basis for the column space of A.

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$$

+2

8. In the blank next to each set $\mathcal{S}$, give the appropriate Roman numeral (one per set), selected from the following.

I. The set *IS* a basis for $\mathbb{R}^3$.
II. The set is not a basis for $\mathbb{R}^3$ because it does not span $\mathbb{R}^3$.
III. The set is not a basis for $\mathbb{R}^3$ because the vectors are not linearly independent.
IV. The set is not a basis for $\mathbb{R}^3$ because it does not span $\mathbb{R}^3$ *AND* the vectors are not linearly independent.

+1

$\mathcal{S}_1 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \right\}$ IV

$\mathcal{S}_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$ II

$\mathcal{S}_3 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ I

$\mathcal{S}_4 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ III