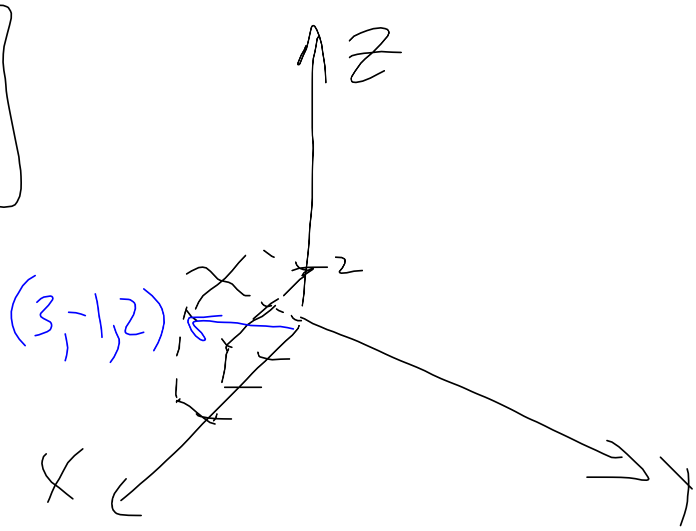
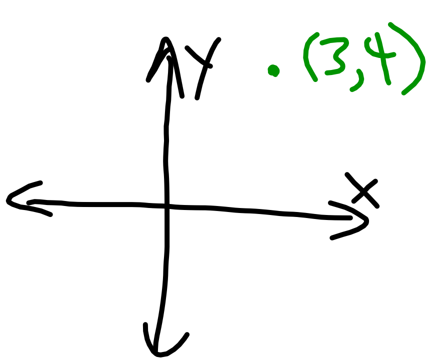
Algebra (+ Calculus)	numbers	functions
Linear Algebra	vectors	transformations (linear transformations)

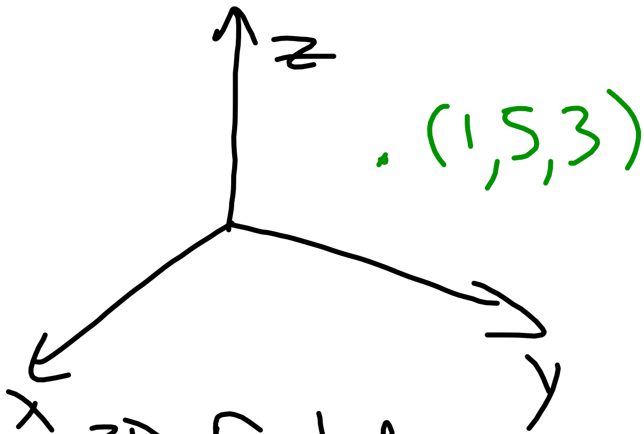
$$\vec{u} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$





2-dimensional Euclidean space

$\mathbb{R}^2$



3D Euclidean space

$\mathbb{R}^3$

$(5, -1, 7, 3)$ is in $\mathbb{R}^4$

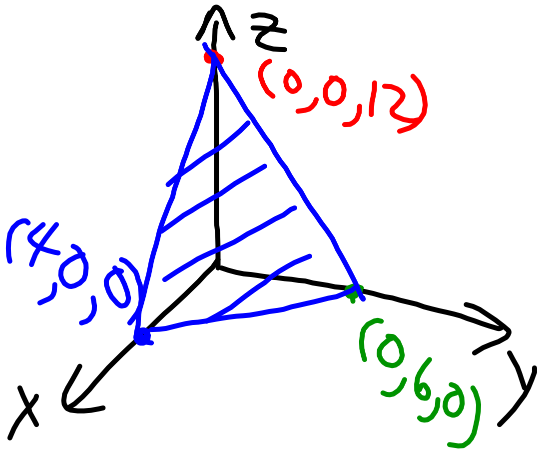
4-tuple

In $\mathbb{R}^3$, the set of points (x, y, z) satisfying $ax + by + cz = d$, a, b, c constants, is a plane.

$ax + by = c$ in $\mathbb{R}^2$

$2x + 5y = 7 \longrightarrow y = -\frac{2}{5}x + \frac{7}{5}$ It's a line!

$3x + 2y + z = 12$



Plane

x-int is 2

y-int is 5

z-int is 4

Equation?

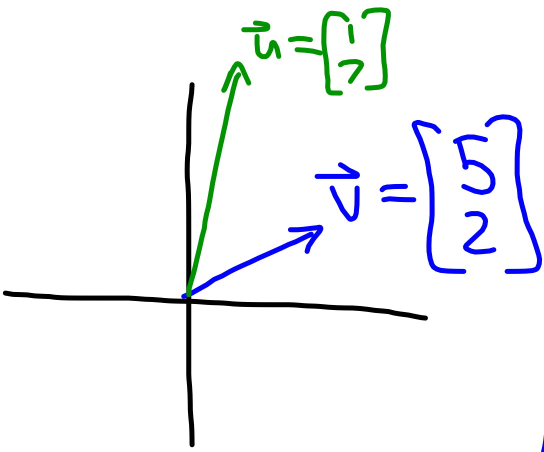
$ax + by + cz = d$

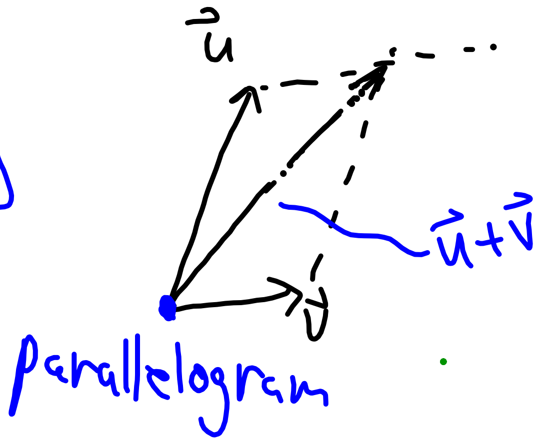
$2a = d$ $10x + 4y + 5z = 20$

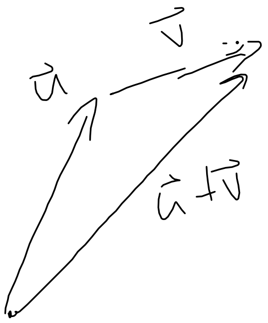
$5b = d$

$4c = d$

$$\vec{u}+\vec{v}=\begin{bmatrix}1\\7\end{bmatrix}+\begin{bmatrix}5\\2\end{bmatrix}=\begin{bmatrix}6\\9\end{bmatrix}$$







tip-to-tail

$$3\vec{v} = \begin{bmatrix} 15 \\ 6 \end{bmatrix}$$

$$c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + c_4\vec{v}_4 + \cdots + c_n\vec{v}_n$$

This is a <u>linear combination</u> of $\vec{v}_1, \vec{v}_2, \vec{v}_3, \ldots$

Huge idea!

Find a linear combination of

$\vec{u}_1 = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$ and $\vec{u}_2 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$ that equals $\begin{bmatrix} 16 \\ 6 \end{bmatrix}$

$$c_1 \begin{bmatrix} -3 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} -3c_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} 2c_2 \\ 5c_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} -3c_1 + 2c_2 \\ c_1 + 5c_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$-3c_1 + 2c_2 = 16$$
$$c_1 + 5c_2 = 6$$

$$c_1 = -4$$
$$c_2 = 2$$

$$f(x) = \underbrace{1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots}_{\text{linear combination of } 1, x, x^2, x^3, \ldots}$$