Solve #

$$x_1 - 3x_2 - x_3 + 6x_4 = 11$$
$$x_1 - 3x_2 + x_3 - 2x_4 = 9$$
$$2x_1 - 6x_2 - x_3 + 8x_4 = 21$$

looking for the intersection of 3 3D planes in $\mathbb{R}^4$

Use your calculator!

Office

T 11-12, 1-2
other if asked
W 11-12, 1-3

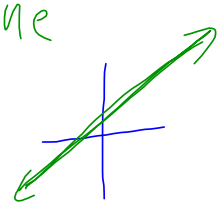
$$\begin{bmatrix} 1 & -3 & -1 & 6 & 11 \\ 1 & -3 & 1 & -2 & 9 \\ 2 & -6 & -1 & 8 & 21 \end{bmatrix} \begin{matrix} -R_1+R_2 \to R_2 \\ \Longrightarrow \\ -2R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & -3 & -1 & 6 & 11 \\ 0 & 0 & 2 & -8 & -2 \\ 0 & 0 & 1 & -4 & -1 \end{bmatrix}$$

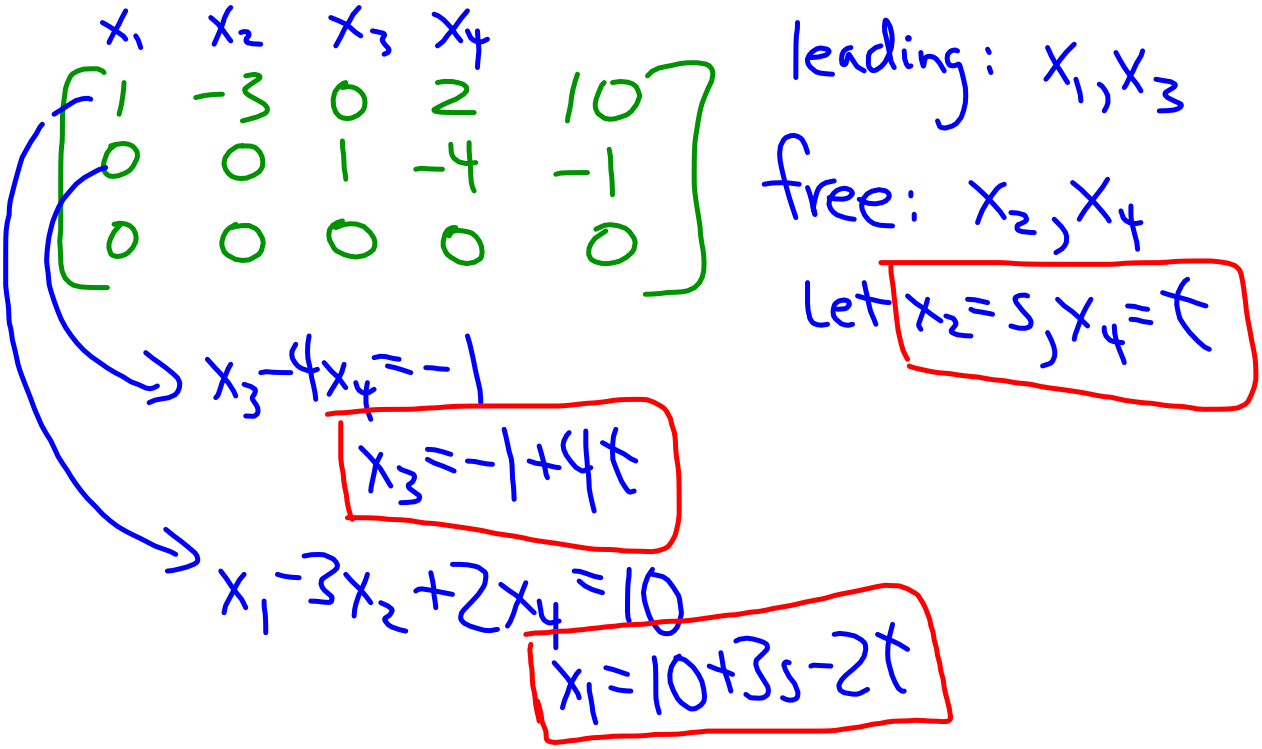
$$R_2+(-2)R_3 \to R_3$$

$2x - y = 3$ 1-D plane ~~line~~ in $\mathbb{R}^2$

$2x - y + 5z = 1$ 2D-plane in $\mathbb{R}^3$

$x_1 - 3x_2 - x_3 + 6x_4 = 11$ 3D-plane in $\mathbb{R}^4$




x_1 x_2 x_3 x_4
1 -3 0 2 10
0 0 1 -4 -1
0 0 0 0 0
leading: x_1, x_3
free: x_2, x_4
let $x_2 = s, x_4 = t$
$x_3 - 4x_4 = -1$
$x_3 = -1 + 4t$
$x_1 - 3x_2 + 2x_4 = 10$
$x_1 = 10 + 3s - 2t$

$x_1 = 10 + 3s - 2t$

$x_2 = s$

$x_3 = -1 + 4t$

$x_4 = t$

General solution

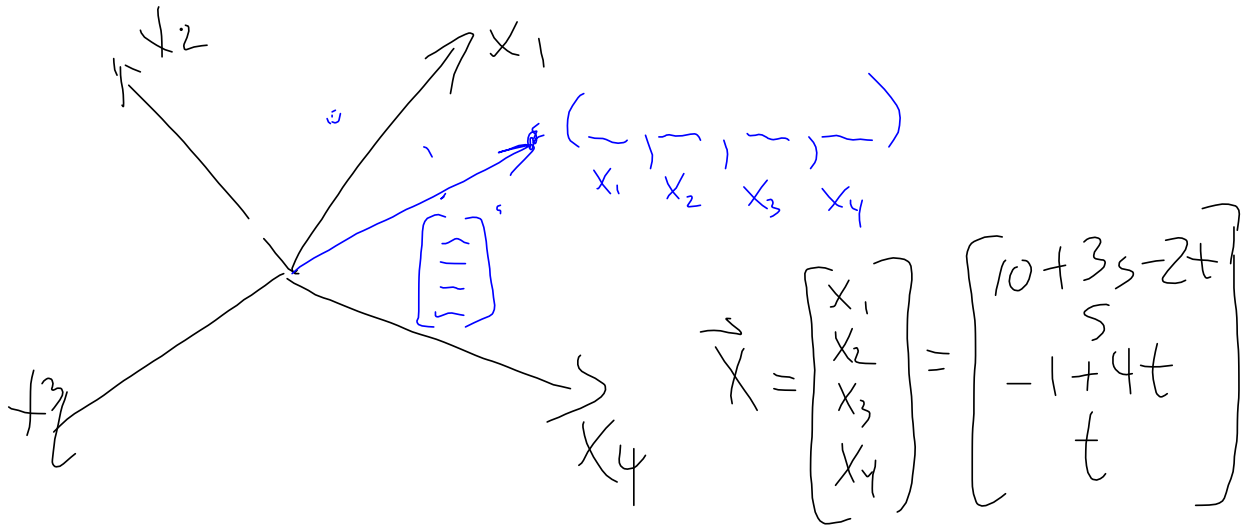
Particular solutions

$s=0$, $t=0$ $(10, 0, -1, 0)$

$s=0$, $t=1$ $(8, 0, 3, 1)$

etc.

$(10+3s-2t, s, -1+4t, t)$


x_2
x_1
x_3
x_4
$(\_, \_, \_, \_)$
x_1 x_2 x_3 x_4
$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10+3s-2t \\ s \\ -1+4t \\ t \end{bmatrix}$

$$\vec{x} = \begin{bmatrix} 10+3s-2t \\ 5 \\ -1+4t \\ t \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3s \\ 5 \\ 0 \\ 6 \end{bmatrix} + \begin{bmatrix} -2t \\ 0 \\ 4t \\ t \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 10 \\ 0 \\ -1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} -2 \\ 0 \\ 4 \\ 1 \end{bmatrix}$$

s, t

2D plane through origin in $\mathbb{R}^4$

$x_1 \ldots x_4$

line through origin

$\vec{x} = \vec{p} + t\vec{q}$

line through tip of $\vec{p}$

$$x_1 - 3x_2 - x_3 + 6x_4 = 11$$
$$x_1 - 3x_2 + x_3 - 2x_4 = 9$$
$$2x_1 - 6x_2 - x_3 + 8x_4 = 21$$

$$\begin{bmatrix} x_1 - 3x_2 - x_3 + 6x_4 \\ \vdots \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 21 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_1 \\ 2x_1 \end{bmatrix} + \begin{bmatrix} \quad \end{bmatrix} + \begin{bmatrix} \quad \end{bmatrix} + \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 21 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + x_4 \begin{bmatrix} 6 \\ -2 \\ 8 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 21 \end{bmatrix}$$

linear combination form of system

Find a l.c. of $\begin{bmatrix}\end{bmatrix}, \begin{bmatrix}\end{bmatrix}, \begin{bmatrix}\end{bmatrix}, \begin{bmatrix}\end{bmatrix}$

equalling $\begin{bmatrix}11\\9\\21\end{bmatrix}$