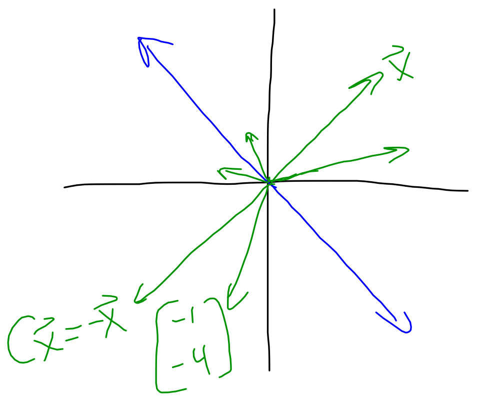
① C is a reflection across $y=-x$.

a) Find $C\begin{bmatrix}4\\1\end{bmatrix}=\begin{bmatrix}-1\\-4\end{bmatrix}$ $\vec{x}=\begin{bmatrix}-2\\1\end{bmatrix}$

b) Find $\vec{x}$ such that $C\vec{x}=\begin{bmatrix}-1\\2\end{bmatrix}$

c) Find $\vec{x}$ such that $C\vec{x}=-\vec{x}$. $\vec{x}=\begin{bmatrix}a\\a\end{bmatrix}$


$\vec{x}$
$c\vec{x} = -\vec{x}$
$\begin{bmatrix} -1 \\ -4 \end{bmatrix}$

(2) Give an $\vec{x}$ such that $R_{90°}\,\vec{x} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

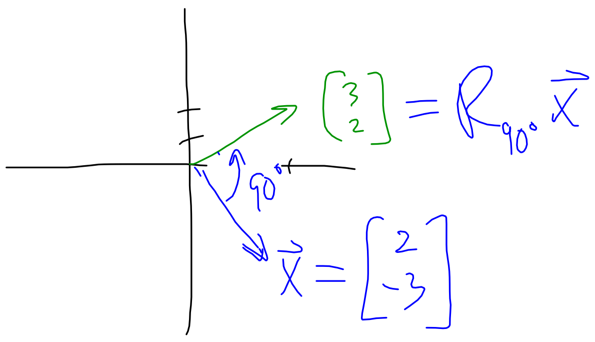
(3) Let P be the projection onto the x-axis.

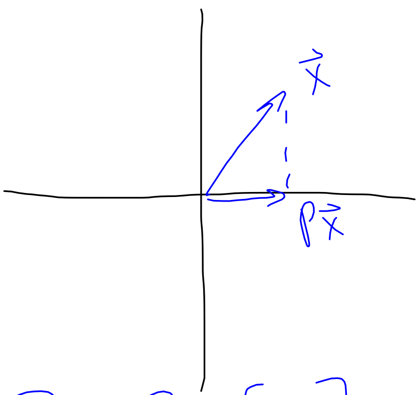
a) Give $P\begin{bmatrix} -5 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \end{bmatrix}$

b) Give an $\vec{x}$ not on the x-axis such that $P\vec{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\quad \vec{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ or $\vec{x} = \begin{bmatrix} 2 \\ 9 \end{bmatrix}$

c) Is P invertible? No

d) Give the matrix for P.





d) $P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ 0 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Longrightarrow P^{-1} = \frac{1}{0 - 0}\begin{bmatrix} \sim \\ \sim \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Longrightarrow A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Assn 5:

① $A\vec{x}=\vec{b}$ A is square

$\boxed{\det(A)\neq 0 \iff A \text{ is invertible}}$

If $\det(A)\neq 0$, $A\vec{x}=\vec{b}$ has a unique sol.

If $\det(A)=0$, $A\vec{x}=\vec{b}$ has no sol or inf many sols.

Homogeneous System

$A\vec{x}=\vec{0}$ always has at least on sol, $\vec{x}=\vec{0}$.

$$A\vec{x} = \vec{b}$$

$$A^{-1}(A\vec{x}) = A^{-1}\vec{b}$$ Memorize

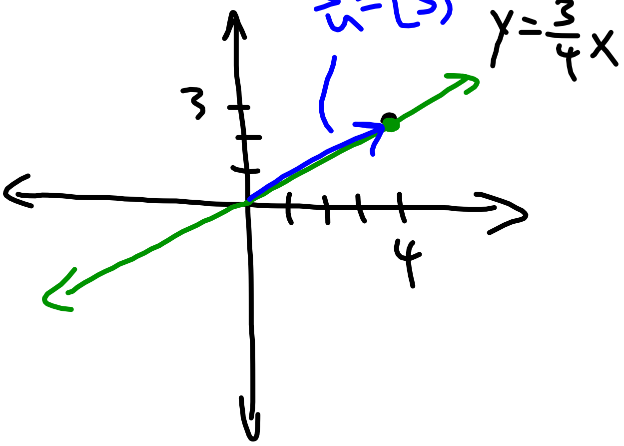
$$(A^{-1}A)\vec{x} = A^{-1}\vec{b}$$

$$I\vec{x} = A^{-1}\vec{b}$$

$$\vec{x} = A^{-1}\vec{b}$$

Assn 6, #4

$\vec{u} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$



$C\vec{u} = \vec{u}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{7}{25} & \frac{24}{25} \\ \frac{24}{25} & -\frac{7}{25} \end{bmatrix}$$

$4,3$

$4^2+3^2=25$

$$C = \begin{bmatrix} \frac{a^2-b^2}{a^2+b^2} & \frac{2ab}{a^2+b^2} \\ \frac{2ab}{a^2+b^2} & \frac{b^2-a^2}{a^2+b^2} \end{bmatrix}$$

(a,b) $a^2-b^2=a+b$

$(a+b)(a-b)=a+b$

$a-b=1$

$$4a+3b=4$$
$$-3a+4b=3$$

$$a=\frac{\begin{vmatrix}4 & 3\\ 3 & 4\end{vmatrix}}{\begin{vmatrix}4 & 3\\ -3 & 4\end{vmatrix}}=\frac{4^2-3^2}{4^2+3^2}=\frac{7}{25}, \quad b=\frac{\begin{vmatrix}4 & 4\\ -3 & 3\end{vmatrix}}{\begin{vmatrix}4 & 3\\ -3 & 4\end{vmatrix}}=\frac{12+12}{4^2+3^2}=\frac{24}{25}$$

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad B = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A(A\vec{x}) = (AA)\vec{x} = A^2\vec{x}$$

$$A^2 = \begin{bmatrix} & \\ & \end{bmatrix}\begin{bmatrix} & \\ & \end{bmatrix}$$

$$= \begin{bmatrix} \sim & \sim \\ \sim & \sim \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix} = B$$