$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

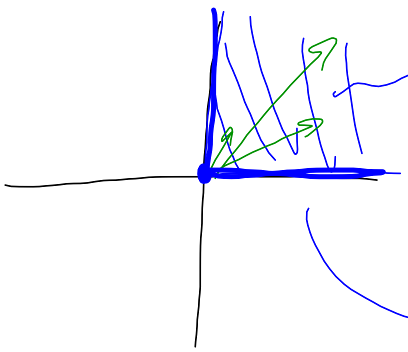
1. Show that $\vec{u} = \begin{bmatrix} -4 \\ 7 \\ 1 \end{bmatrix}$ is in span(S).

2. Show that $\vec{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is in span(S)

$$1 \quad -11\begin{bmatrix}1\\0\\0\end{bmatrix}+6\begin{bmatrix}1\\1\\0\end{bmatrix}+1\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}-4\\7\\1\end{bmatrix}$$

$$a-b\begin{bmatrix}1\\0\\0\end{bmatrix}+b-c\begin{bmatrix}1\\1\\0\end{bmatrix}+c\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$S_1 = \{ \quad \}$ $S_2 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} \right\}$

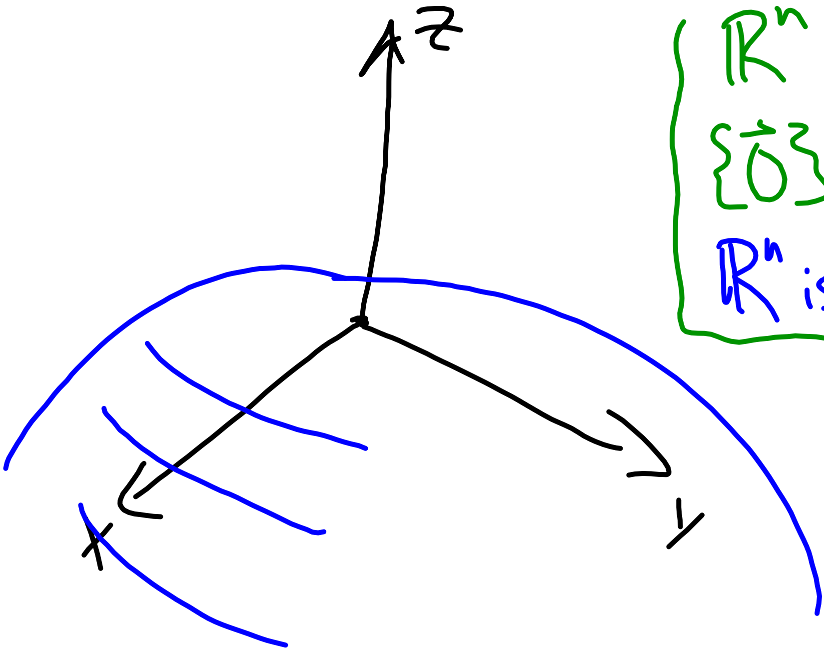


subset of $\mathbb{R}^2$

$A = \{1, 2, 3, 4\}$ $B = \{3, 4\}$

closed under addition, <u>not</u> under scalar mult.

It is a subset, but not a subspace.



$\mathbb{R}^n$

$\{\vec{0}\}$ is a subspace
(the "trivial subspace")

$\mathbb{R}^n$ is a subspace

$S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$ Consider span(S)

$\vec{u}_1, \vec{u}_2 \in$ span(S). Is $\vec{u}_1 + \vec{u}_2$ in span(S)

$\vec{u}_1 = \underbrace{c_1\vec{v}_1 + c_2\vec{v}_2 + \cdots + c_n\vec{v}_n}$, $\vec{u}_2 = \underbrace{d_1\vec{v}_1 + d_2\vec{v}_2 + \cdots + d_n\vec{v}_n}$.

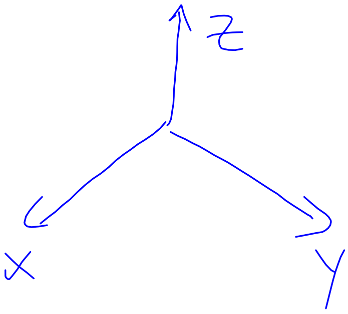
$\vec{u}_1 + \vec{u}_2 = \qquad + \qquad$

$= c_1\vec{v}_1 + d_1\vec{v}_1 + \cdots c_n\vec{v}_n + d_n\vec{v}_n$

$= (c_1 + d_1)\vec{v}_1 + \cdots + (c_n + d_n)\vec{v}_n$

The span of a set of vectors in $\mathbb{R}^n$ is a subspace of $\mathbb{R}^n$.

A subspace contains the zero vector.



The span of $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ is the xy-plane.

The set $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$ spans the xy-plane.

Given a subset, S of $\mathbb{R}^n$, is it a subspace.

* Is $\vec{0} \in S$? If no, not a subspace.
* Pick a few specific vectors, try adding + multiplying by scalar.
  If not closed under either, not a subspace.
* Prove it is a subspace.
  - Show closed under addition + scalar mult.

    OR

  - Show it is the span of a specific set of vectors.