indep dep

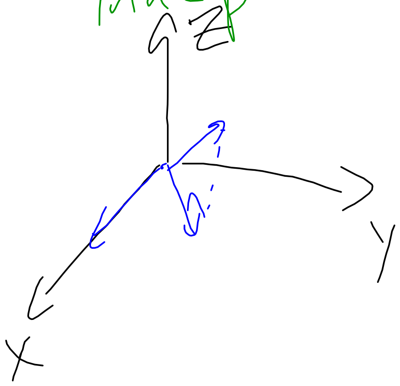
$$S_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}, \quad S_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

$$S_3 = \left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -15 \end{bmatrix} \right\}, \quad S_4 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \right\}$$

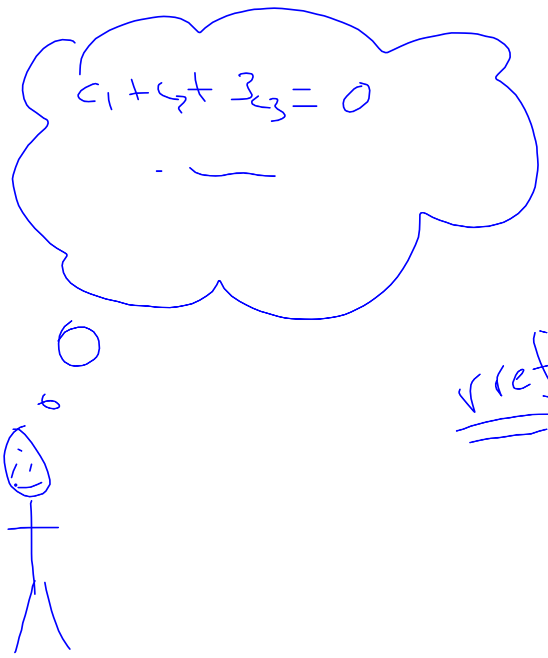
dep

$$S_5 = \left\{ \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}, \quad S_6 = \left\{ \begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix} \right\}$$

indep indep



$$c_1\begin{bmatrix}1\\2\\3\end{bmatrix} + c_2\begin{bmatrix}1\\1\\1\end{bmatrix} + c_3\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & 3 & 0 \\ 2 & 1 & 2 & 0 \\ 3 & 1 & 1 & 0 \end{bmatrix}$$

$$\xrightarrow{\text{rref}} \begin{array}{c} \begin{matrix} c_1 & c_2 & c_3 & \end{matrix} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$c_3 = 1$

$c_2 + 4c_3 = 0$

$c_2 = -4$

$c_1 = 1$

$$1\begin{bmatrix}1\\2\\3\end{bmatrix} - 4\begin{bmatrix}1\\1\\1\end{bmatrix} + 1\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1\\2\\3\end{bmatrix} = 4\begin{bmatrix}1\\1\\1\end{bmatrix} - \begin{bmatrix}3\\2\\1\end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

<u>Matrix A</u> has 2 subspaces:

* column space $\quad col(A)$
* null space $\quad null(A)$

$A\vec{x} = \vec{b}$

$A\vec{x} = \vec{0}$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 \\ 4 & 1 & 0 & 2 \end{bmatrix} \qquad \vec{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 4 \\ 7 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} -1 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

$\vec{u}_1$: $\notin \text{col}(A)$, $\notin \text{null}(A)$

$\vec{u}_2$: $\in \text{col}(A)$, $\notin \text{null}(A)$

$\vec{u}_3$: $\notin \text{col}(A)$, $\in \text{null}(A)$

$A\vec{u}_3 = \vec{0}$, so

$\vec{u}_3 \in \text{null}(A)$