$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

Basis

Linear Transformations

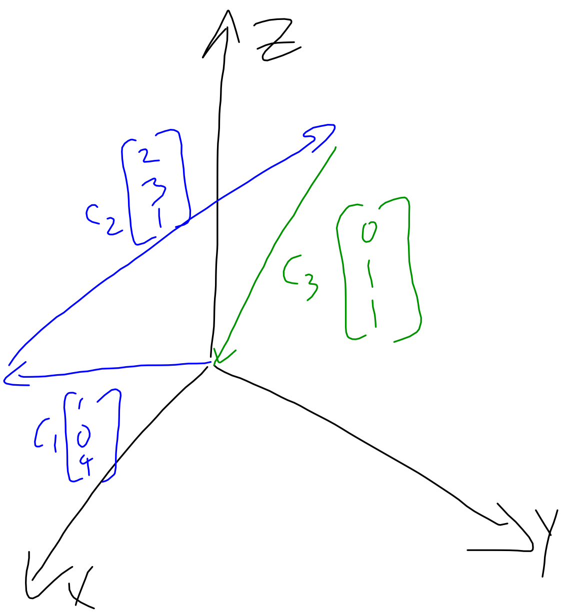
1. Is this set linearly independent?
2. Does the set span $\mathbb{R}^3$?

$$c_1 \begin{bmatrix}1\\0\\4\end{bmatrix} + c_2 \begin{bmatrix}2\\3\\1\end{bmatrix} + c_3 \begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

Yes, the set is linearly independent.

$c_1 + 2c_2 + 0c_3 = 0$

$$\Rightarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \xRightarrow{\text{rref}} \begin{bmatrix}1 & 0 & 0\\ & 1 & 0\\ 0 & 1 & 0\end{bmatrix}$$


Z
Y
X
c2 [2 3 1]
c3 [0 1 1]
c1 [1 0 4]

$$c_1\begin{bmatrix}1\\0\\4\end{bmatrix}+c_2\begin{bmatrix}2\\3\\1\end{bmatrix}+c_3\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&2&0&a\\0&3&1&b\\4&1&1&c\end{array}\right]\xrightarrow{\text{rref}}\left[\begin{array}{ccc|c}1&0&0&\#\\0&1&0&\#\\0&0&1&\#\end{array}\right]$$

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$ is a <u>basis</u> for $\mathbb{R}^3$

1. spans $\mathbb{R}^3$ → we make anything in $\mathbb{R}^3$ out of these
2. It is independent → we can only do it one way

Standard basis for $\mathbb{R}^3$ is

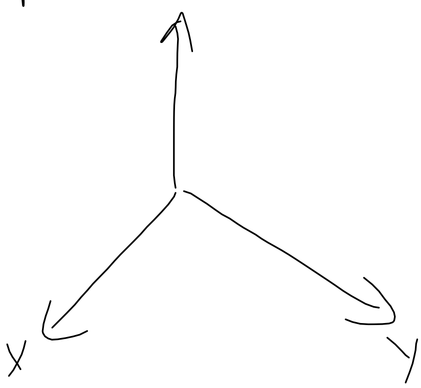
$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$$

$$\langle 3, 5, 1 \rangle = 3\vec{i} + 5\vec{j} + 1k$$

The xy-plane is a subspace of $\mathbb{R}^3$

Give a basis for the xy-plane:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$



Give another basis for the xy-plane:

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \right\} \qquad \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} \right\}$$

$$S = \left\{ s\begin{bmatrix}1\\1\\1\end{bmatrix} + t\begin{bmatrix}1\\2\\3\end{bmatrix} \right\} \underbrace{\subseteq}_{\text{subset}} \mathbb{R}^3$$

Is S a subspace?

S is $\operatorname{span}\left(\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\2\\3\end{bmatrix}\right\}\right)$ so, by Cor. 4.3.3

What is a basis for S?

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}, \quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$$

is also a basis for S.