

$$A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 2 & -2 & -1 & 3 \\ -1 & 1 & -1 & 0 \end{bmatrix}, \vec{u}_1 = \begin{bmatrix} 4 \\ 9 \\ -6 \end{bmatrix}, \vec{u}_2 = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix},$$

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ -2 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ - \\ - \\ 1 \end{bmatrix}$$

① Are either of  $\vec{u}_1, \vec{u}_2 \in \text{col}(A)$

② Are either of  $\vec{v}_1, \vec{v}_2 \in \text{null}(A)$ ?

$$c_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} + c_4 \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ -6 \end{bmatrix}$$

$$\left[ A \mid \begin{array}{c} 4 \\ 9 \\ -6 \end{array} \right]$$

$$\begin{array}{cccc|c} c_1 & c_2 & c_3 & c_4 & \\ \hline 1 & -1 & 0 & 1 & 5 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

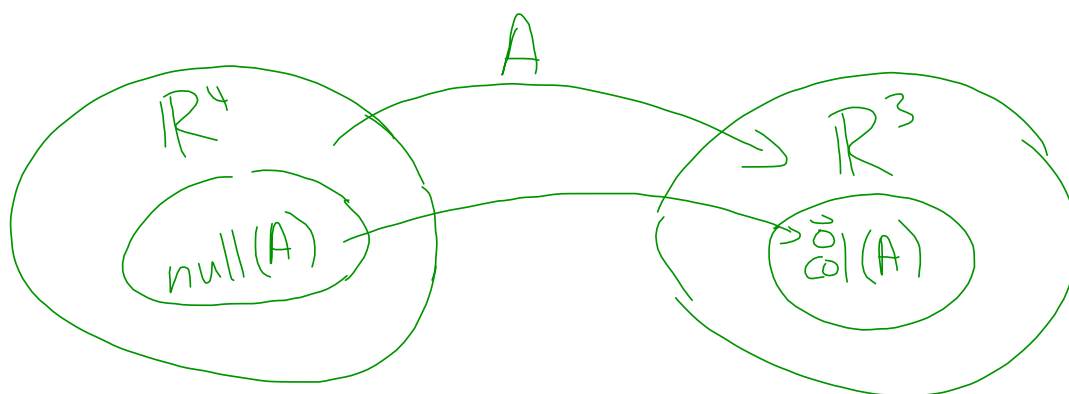
$c_1 = 5$   
 $c_2 = 0$   
 $c_3 - c_4 = 1$   
 $c_4 = 0$

$c_2, c_4$  are free

$$\text{yes, } \begin{bmatrix} 4 \\ 9 \\ -6 \end{bmatrix} \in \text{col}(A), \quad 5 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ -6 \end{bmatrix}$$

$$A\vec{v}_1 = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} \vec{v}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \neq \vec{0} \text{ so } \vec{v}_1 \notin \text{null}(A)$$

$$A\vec{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \implies \vec{v}_2 \in \text{null}(A)$$



Vector space  $V$  (think  $\mathbb{R}^2, \mathbb{R}^3$ , etc.)

$W$  is a subspace of  $V$

A set  $\mathcal{B}$  is a basis for  $W$

if

\*  $\mathcal{B}$  spans  $W$

\*  $\mathcal{B}$  is independent

Facts: ① Any set of less than  $n$  vectors cannot span  $\mathbb{R}^n$ .

② Any set of more than  $n$  vectors in  $\mathbb{R}^n$  must be linearly dependent.

B.  $S$  is a basis for  $\mathbb{R}^2$  or  $\mathbb{R}^3$ .

C.  $S$  does not span and is dependent

E.  $S$  is dependent, but does span

D.  $S$  does not span, but are independent