$$A=\begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Are either of $\vec{u}_1=\begin{bmatrix} 3 \\ 3 \\ -3 \\ 5 \end{bmatrix}$ or $\vec{u}_2=\begin{bmatrix} 1 \\ 4 \\ -2 \\ 1 \end{bmatrix}$ in $\text{col}(A)$?

If so, show why.

Are either of $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \\ 2 \end{bmatrix}$ or $\vec{v}_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \\ -4 \\ 1 \end{bmatrix}$ in null(A)?

$A\vec{x} = \vec{0}$?

If so, show why.

Yes

$\boxed{A\vec{v}_2 = \vec{0}}$

$$\begin{array}{c} \begin{array}{ccccc} 1 & 1 & 1 & 1 & -2 \end{array} \\ A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix} \end{array} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 \\ 4 \\ -4 \\ 6 \end{bmatrix} \in \text{col}(A)$$

$$\mathcal{B} := \left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\} \text{ is a basis for } \mathrm{col}(A)$$

$$4 \quad -3 \quad -7$$

$$A\vec{x} = \vec{b}$$

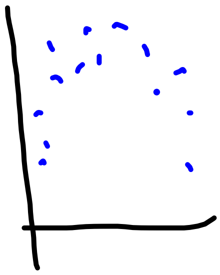
If $\vec{b} \in \mathrm{col}(A)$, there is at least one solution.

$$\begin{bmatrix} x & x \\ x & x \\ x & x \\ x & x \\ x & x \\ x & x \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} x \\ x \\ x \\ x \\ x \\ x \end{bmatrix}$$

$$A \quad \vec{x} = \vec{b}$$

$$\vec{b} \notin col(A)$$

$$\bar{x} = \overset{2\times 2}{(A^T A)^{-1}} \underbrace{A^T \vec{b}}_{\begin{bmatrix} x \\ x \end{bmatrix}}$$

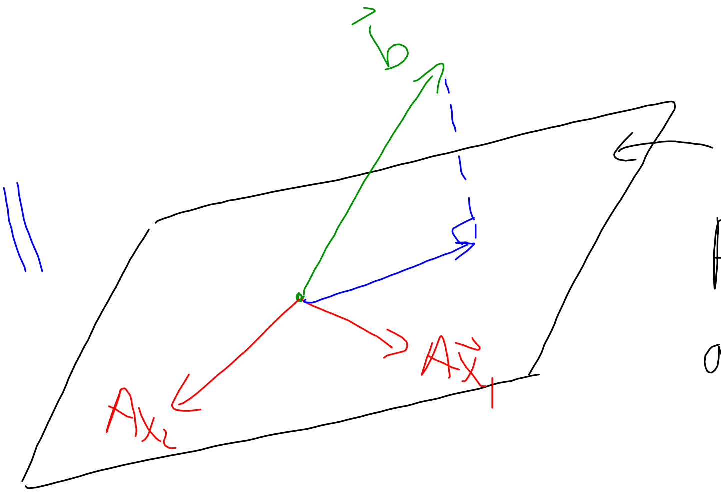


$$y = ax^2 + bx + c$$

$\mathbb{R}^6$

minimize

$\|\vec{b} - A\vec{x}\|$



col(A)

$A\vec{x}$ for all $\vec{x} \in \mathbb{R}^2$

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix} \xRightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$A\vec{x} = \vec{0}$

$A\vec{x} = \vec{b}$

free

$$\begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ \left[\begin{matrix} 1 & 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}\right] \\ & & s & & t & \end{array}$$

$\rightsquigarrow x_2 + 2x_3 + 3x_5 = 0$

$x_4 + 4x_5 = 0$

$x_1 = -s + t$

$x_2 = -2s - 3t$

$x_3 = s$

$x_4 = -4t$

$x_5 = t$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -s+t \\ -2s-3t \\ s \\ -4t \\ t \end{bmatrix} = \begin{bmatrix} -s \\ -2s \\ s \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} t \\ -3t \\ 0 \\ -4t \\ t \end{bmatrix} = s\begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

$$\mathcal{B} = \left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right\} \text{ is a basis null}(A)$$